Follow the directions for each section carefully. NOTE: Section III has a choice of problems.

The total time allotted for this day is 4 hours. Scores on questions will be weighted in proportion to the allotted time.

Start each numbered problem on a new page. Put your name on every page.

When you do not have time to put answers into forms that satisfy you, indicate specifically how you would proceed if more time were available. If you do not attempt a particular problem, write on a sheet of paper "I have not attempted Problem ____ " and sign your name.

GOOD LUCK!
SECTION I - PLASMA WAVES

COMPLETE ALL PROBLEMS IN THIS SECTION. NUMBER EACH PROBLEM CORRECTLY AS SECTION #, PROBLEM #.
SECTION I - PROBLEM 1

WAVE THEORY - 30 mins.

A small disturbance in a plasma evolves as

\[ \psi(x,t) = \frac{1}{\sqrt{2\pi}} \int dk \psi(k)e^{ikx-i\omega(k)t} \]

where \( \omega = \omega(k) \) is the dispersion relation.

3 mins.

a) If the disturbance at \( t = 0 \) is strongly localized,

\[ \psi(x,0) \sim \delta(x) \]

what is \( \psi(k) \)?

4 mins.

b) Say that \( \omega(k) = \omega_r(k) + i\gamma(k) \) where \( \omega_r \) and \( \gamma \) are real, for \( \text{real} k \), but \( |\gamma| \ll |\omega| \). Locate the peak of the disturbance for \( t > 0 \).

12 mins.

c) Next, expand \( \omega(k) = \omega(k_o) + (k - k_o)\omega'(k_o) + (1/2)(k - k_o)^2\omega''(k_o) + \ldots \), where \( k_o \) is real and chosen such that \( \gamma'(k_o) = 0 \). Assuming that contributions in the neighborhood of \( k = k_o \) are the only important ones, find \( \psi(x,t) \).
4 mins.

d) For a fixed value of t, what is the pulse shape in space?

7 mins.

e) Based on your answers to (c) and (d), what is the criterion for an absolute instability, i.e., for exponential growth of $\psi(0,t)$?
SECTION I - PROBLEM 2

Ion-Acoustic Instability - 40 mins.

Consider a plasma consisting of cold ions and drifting electrons with one-dimensional distribution

\[ F(v_z) = \frac{\alpha}{\pi[(v_z - v_o)^2 + \alpha^2]} \]

where \( v_o << \alpha \).

15 mins.

a) Derive the dispersion relation for electrostatic oscillations with \( \vec{k} \) parallel to \( \hat{z} \).

15 mins.

b) Show (for example using a Penrose diagram) that there are unstable waves for \( v_o \neq 0 \).

10 mins.

c) Describe qualitatively how the waves and the electron distribution evolve in time.
SECTION I - PROBLEM 3

Electromagnetic Waves - 15 mins.

Consider an unmagnetized plasma near thermal equilibrium. Using a cold fluid description for the electrons (ignore the ions), derive a linear dispersion relation for the electromagnetic waves. Show that a zero frequency magnetic perturbation exists in addition to the high frequency oscillations (Hint: retain $\nu_{ei}$ in the momentum equation).
SECTION I - PROBLEM 4

Wave Stability - 10 mins.

In analyzing a one-dimensional problem in which a plasma is excited at constant frequency $\omega = \omega_0$ (real), you determine that $D(\omega_0, k_z)$ has a root with $\text{Im}(k_z) > 0$. Describe how you might systematically determine whether this root corresponds to an unstable or stable wave.
SECTION II - MHD THEORY

COMPLETE ALL PROBLEMS IN THIS SECTION. NUMBER EACH PROBLEM CORRECTLY AS SECTION #, PROBLEM #.
SECTION II - PROBLEM 1

Global Inequality - 30 mins.

Show that in any finite closed magnetostatic system with B tangent to the boundary, one must have the inequality

\[ \left( \frac{B^2}{8\pi} + p \right)_{\text{max}} \geq [3p + \frac{B^2}{8\pi}]_{\text{av}}. \]

where the maximum on the left refers to the maximum over the bounding surface while the average on the right is the volume average.

HINT: Write the equation of motion as

\[ \rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = - \nabla \cdot \mathbf{T} \]

set \( \mathbf{v} = 0 \), and multiply by \( \mathbf{r} \) and integrate over the volume.
SECTION II - PROBLEM 2

Stable MHD Continuous Spectrum in a Cold Diffuse Linear Pinch - 45 mins.

Let us consider fixed-boundary modes in a cold diffuse linear pinch (or general screw pinch, see the accompanying figure).

\[
B = B_z + B_\phi(r), \quad B_z = \text{const.}
\]

That is, we let the radial component of the MHD fluid displacement be

\[
\xi(r) \exp(i m \phi - i k z - i \omega t),
\]

\[
k = n/R_o \quad \text{and} \quad \xi(r = a) = 0.
\]

Noting that the plasma is cold, \( P_o = 0 \), and after eliminating the compressional Alfvén waves, the perturbed Lagrangian of the MHD fluid; \( \delta L \), is given as follows

\[
\delta L = \delta K - \delta W,
\]

\[
\delta K = 2\pi^2 R_o \int_0^a r dr \omega^2 \rho_o \{\xi^2 + [(r \xi)'/m]^2\}
\]
with $\rho_o$ being the mass density, denotes the fluid kinetic energy, $A' = dA/dr$, and

$$\delta W = \frac{\pi B^2}{2R_o} \int_0^a \frac{dr}{r} f^2 \left[ r^2 (\xi')^2 + (m^2 - 1) \xi^2 \right]$$

denotes the potential energy where $f = 1/q - n/m$ and $q(r) = rB_Z/R_o B_\theta$.

1) Extremizing $\delta L$ to obtain the Euler-Lagrangean equation which is a second-order ordinary differential equation for $\xi(r)$.

2) Note that the coefficient for the $d^2\xi/dr^2$ term is of the form

$$\omega^2 g(r) - h(r)f^2(r),$$

where $g(r)$, $h(r)$ are functions of $r$ and $f(r)$ is given above. Thus, for $\omega^2 = \omega_A^2(r) = h(r)f^2(r)/g(r)$, we could have singular solutions and $\omega_A^2(r)$ corresponds to the MHD continuous spectrum.

2a) Show that $\omega_A^2(r) > 0$, i.e., we have a stable continuous spectrum. Identify $\omega_A^2$ with one branch of the MHD waves.

2b) Let $\omega^2 = \omega_A^2(r)$ at $r = r_A$. Derive the singular solution of $\xi(r)$ in terms of the variable $x = r - r_A$. 
SECTION II - PROBLEM 3

Energy Principle in Mirror Machines - 10 mins.

In dealing with ideal MHD stability in tokamaks, we often can minimize the sound-wave compressional energy

$$\delta W_{f2} = \frac{1}{2} \int \text{d}^3x \, \gamma \, P_0 |\nabla \cdot \delta \xi|^2$$

to zero, where $\gamma$ is the ratio of specific heat, $P_0$ is the equilibrium pressure and $\delta \xi$ is the fluid displacement vector. Can we do the same minimization in mirror machines? Substantiate your answer.
SECTION III - APPLIED MATH

IN THIS SECTION, CHOOSE EITHER PROBLEM 1 - COLD FUSION, OR 2 - MATRIX ANALYSIS, AND THEN DO PROBLEM 3. BE SURE TO GIVE SECTION # AND PROBLEM #.
SECTION III - PROBLEM 1 (DO THIS PROBLEM OR #2 AND #3)

Cold Fusion - 45 mins.

You don’t need any special knowledge for this question, just some asymptotics. If you run low on time just describe what you would do if you had more time.

The rate of nuclear reaction at low energy is proportional to $|\psi(\rho)|^2$ where $\psi(r)$ is the wave function for the nuclei with relative separation $r$ and $\rho$ is the characteristic range of the strong force.

In a $D_2$ molecule, the nuclear wave function $\psi(r)$ obeys an equation like,

$$\left(\frac{m_e}{2M}\right) \frac{d^2 u}{dr^2} + Eu - \left[\frac{1}{r} - \frac{2}{(rr_o)^{1/2}}\right] u = 0$$

(1)

where: $M$ = mass of deuterium, $m_e$ = mass of electron, $u = r\psi(r)$, $1/r$ is the repulsive potential between D atoms, $-2/(rr_o)^{1/2}$ is a model of the electron screening which "binds" the molecule together. Lengths are measured in Bohr radii ($a_o = \frac{n^2}{me^2}$) and energy in units of $e^2/a_o$. Note that $m_e/M \sim 1/4000 << 1.$

5 mins.

a) Give a rough plot of the potential.

10 mins.

b) Show that the lowest energy eigenstate is given approximately by
\[ u = u_0 \exp \left( - \left( \frac{r - r_0}{2} \right)^2 \sigma \right) \]  

(2)

where: \( \sigma = \frac{1}{2} \left( \frac{M}{m_r r_0^3} \right)^{1/2} \) and \( E + \frac{1}{r_0} = \delta E = \frac{1}{2} \left( \frac{m_e}{M r_0^3} \right)^{1/2} \)

and normalization of \( \psi \) gives \( u_0 = \frac{1}{\sqrt{4\pi}} \left( \sigma \right)^{1/4} \).

For what values of \( r \) is Eq. (2) valid?

[HINT: Lowest eigenfunction of \( d^2y/dx^2 + (E - x^2) y = 0 \) is \( y = y_0 e^{-x^2/2}, E = 1 \)]

30 mins.

c) For \( \frac{r - r_0}{r_0} = 0(1) \) we may use the W.K.B. formulae: Hence

\[
U_{W.K.B.} = \frac{u_0}{\left[ \frac{1}{r} + \frac{1}{r_0} - 2/(r r_0)^{1/2} - \delta E \right]^{1/4}}
\]

\[
\exp - \sqrt{\frac{M}{m_e}} \int_r^{r_1} \left[ \frac{1}{r} + \frac{1}{r_0} - 2/(r r_0)^{1/2} - \delta E \right]^{1/2} dr
\]

(3)

with \( r_1 \) chosen to be

\[
\frac{1}{r_1} + \frac{1}{r_0} - 2/(r_1 r_0)^{1/2} = \delta E
\]
Near \( r = 0 \) the W.K.B. formula breaks down, however, we will not be concerned with that here. The other W.K.B. solution is ruled out by boundary conditions at the origin.

**FIND** \( \bar{u}_0 \) in terms of \( u_0 \) by matching the W.K.B. solution of Eq. (3) to the eigenfunction of Eq. (2). Explain carefully where the region of "overlap" occurs.
SECTION III - PROBLEM 2 (DO NOT DO THIS PROBLEM IF YOU DID PROB. 1)

Matrix Analysis - 45 mins.

Consider the two-by-two complex matrix

\[
M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}
\]  \hspace{1cm} (1)

Under what conditions on \(a, b, c,\) and \(d\) can it be diagonalized; that is when can a nonsingular matrix \(S\) be found such that

\[
S^{-1} MS = D
\]  \hspace{1cm} (2)

is diagonal? Carry this out as follows:

15 mins.
1) Show that \(S\) exists if two independent eigenvectors of \(M\) exist and that the two columns of \(S\) are these eigenvectors. Show that if two independent eigenvectors exist, then the matrix \(S\) with these eigenvectors for its columns is nonsingular and satisfies (2).

10 mins.
2) What is the characteristic function \(f(\lambda) = \det(M - \lambda I)\)? Show that when \(f\) has two unequal roots \(M\) can be diagonalized. What is the condition on \(a, b, c\) and \(d\) that this is the case?
10 mins.

3) Show if \( f(\lambda) \) has two equal roots \( M \) may be diagonalized only if \( a = d \) and \( b = c = 0 \).

10 mins.

4) Answer the question at the beginning as to what two-by-two matrices can be diagonalized. Show that a Hermitian matrix can always be diagonalized.
SECTION III - PROBLEM 3

Asymptotics - 15 mins.

Find the asymptotic behavior of $y$ as $x \to \infty$

$$\frac{d^2y}{dx^2} - e^x y = 0$$
Follow the directions for each section carefully. NOTE: Section V has a choice of problems.

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GOOD LUCK!
SECTION IV - KINETIC THEORY AND TRANSPORT

DO ALL PROBLEMS IN THIS SECTION. NUMBER EACH PROBLEM CORRECTLY AS SECTION #, PROBLEM #.
SECTION IV – PROBLEM 1

Slowing Down – (25 points)

Consider the slowing down and scattering of superthermal electrons in a neutral electron-ion magnetized plasma. Suppose the ion charge state $Z_i >> 1$. The evolution of the electron distribution function, $f$, is described rather well for $t > 0$ by the following equation:

$$\frac{\partial f}{\partial t} = \Gamma \left[ \frac{1}{v^2} \frac{\partial}{\partial v} f + \frac{1 + Z_i}{2v^3} \frac{\partial}{\partial \mu} (1 - \mu^2) \frac{\partial}{\partial \mu} f \right]$$

where $v$ is the electron speed and $\mu = v_y / v$ is the velocity pitch angle.

(5 pts.) a) Interpret each of the terms in Eq.(*) What approximations were used in obtaining this equation from the Landau collision integral? In what region of velocity space does the equation fail to represent adequately the evolution of the distribution and what terms are neglected? Does this equation conserve particles?

(15 pts.) b) Show that, for appropriate boundary and initial conditions, the solution to this equation, if it exists, is unique.

(5 pts.) c) Suppose that initially the distribution function is maxwellian except for a small bunch of high-velocity electrons with parallel velocities say between 6 and 7 times the thermal velocity and perpendicular velocities say less than the thermal velocity. Use contour plots to sketch the evolution of the nonmaxwellian feature.
Particle Diffusion (15 points)

Consider a slab of fully ionized plasma immersed in an intense \( z \)-directed uniform magnetic field. There are two species of ions, \( a \) type and \( b \) type. For all quantities, \( \partial / \partial z = \partial / \partial y = 0 \), but there is a density gradient in the \( z \)-direction. The plasma slab occupies the space \( 0 < z < L \).

(10 pts.) a) It can be shown that there is no net charge transport across field lines due to two-body collisions. If the collisions were modeled by an approximate collision operator, what property of the approximate collision operator would preserve this result? In addition, the particle transport due to temperature gradients and the heat transport due to density gradients obey certain symmetry relations. What property of the collision operator is responsible for the symmetries in transport coefficients?

(5 pts.) b) What mechanisms exist, then, for such a fully ionized plasma to expel net charge? In particular, do three-body collisions permit charge expulsion?
SECTION IV - PROBLEM 3

Neoclassical Theory - 40 mins.

For a tokamak plasma in the banana regime, neoclassical theory produces the following "Onsager" matrix relationship between the fluxes and forces for electrons:

\[
\begin{bmatrix}
\Gamma_{\text{NEO}} \\
<j\parallel>_{\text{NEO}}
\end{bmatrix}
= 
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\cdot
\begin{bmatrix}
-T_e \frac{dn_o}{dr} \\
n_o E\parallel
\end{bmatrix}
\]

Fluxes \hspace{1cm} Matrix \hspace{1cm} Forces

We consider only the neoclassical particle (\(\Gamma\)) and charge (\(<j\parallel>\)) fluxes. Here, \(V_T = 0\), i.e., \(dP_e/dr = T_e \frac{dn_o}{dr}\), and \(E\parallel\) is the parallel electric field. Identify the matrix elements, \(A_{ij}\), and give a simplified (heuristic type) derivation of each term.
SECTION IV - Problem 4

Quasilinear Theory - 55

[55 total min.] This problem concerns quasilinear theory and anomalous transport. Consider a slab of plasma bounded between \( x = 0 \) and \( x = L \). Assume that the plasma may be described by a fluid temperature \( T(x,t) \) and a random fluid velocity \( \vec{u}(t) \hat{x} \). (Note that the velocity depends only on \( t \); it is spatially uniform.) Nothing varies in the \( y \) or \( z \) directions; the problem is completely one-dimensional. Allowing some classical thermal diffusion described by the constant thermal conductivity \( \kappa \), we shall take as our temperature evolution equation

\[
\frac{\partial}{\partial t} T(x,t) + \vec{u}(t) \frac{\partial T}{\partial x} - \kappa \frac{\partial^2}{\partial x^2} T = 0.
\]  

(1)

We shall assume that the plasma is bounded by thermal reservoirs with infinite heat capacity, so the temperature may be specified to be constant (no fluctuations) on the boundaries. To be specific, take as the boundary conditions

\[ T(0,t) = T_0, \quad T(L,t) = 0. \]

We are going to assume that the velocity and, hence, temperature are turbulent, so that we are interested in such things as the averaged thermal flux, where average means an ensemble average over all possible realizations of the flow. Now in a complete description of the plasma, one should solve a momentum equation to determine the plasma velocity. However, here we shall simply characterize the velocity as a stationary Gaussian random variable, with zero mean and specified two-time correlation function \( C(\tau) \):

\[
\langle \vec{u}(t) \rangle = 0,
C(\tau) = \langle \vec{u}(t + \tau) \vec{u}(t) \rangle = \beta^2 \exp(-|\tau|/\tau_{ac}).
\]

Thus, \( \beta \) is the rms level of the velocity fluctuations and \( \tau_{ac} \) is the autocorrelation time of those fluctuations.

The goal is to determine the size of the average thermal flux flowing through this system in various limits.

(a) [10 min.] There are a number of dimensional parameters in this problem: \( T_0 \), \( L \), \( \kappa \), \( \beta \), and \( \tau_{ac} \). However, the dynamics actually depend on just two dimensionless parameters: a Reynolds-like number \( R \) proportional to the strength but not the rapidity of the velocity fluctuations, and the so-called Kubo number \( K \) proportional to the rapidity of the fluctuations. Give appropriate expressions for \( R \) and \( K \), and write Eq. (1) in a suitable set of dimensionless parameters.

Although you may use this dimensionless equation in the remaining parts, you don’t have to if you got confused here.

(b) [8 min.] Derive the continuity equation for the mean temperature \( (T(x,t)) \), where again the ensemble average is taken over all possible realizations of the velocity field. Distinguish carefully between mean quantities and fluctuations around the mean. Identify
from this equation the general expressions for the classical thermal flux $\Gamma_{cl}$ and the nonlinear contribution $\Gamma_n$ to the flux, in terms of the unknown temperature and velocity fluctuations and/or the mean temperature field. (You don’t have to compute the value of these expressions at this time.) Prove that in steady state the total flux $\Gamma(x) = \Gamma_{cl}(x) + \Gamma_n(x)$ is independent of $x$.

(c) [7 min.] When the velocity fluctuations are sufficiently rapid, quasilinear theory is appropriate. What form do you expect for the continuity equation in this limit, in terms of some as-yet unknown quasilinear diffusion coefficient $D_q$? (Explain your intuition.) What is the steady-state solution of the resulting continuity equation for $(T(x))$? Does this solution depend on $D_q$? Why or why not?

(d) [15 min.] Still considering the quasilinear limit, calculate $D_q$ explicitly. Show all steps. It’s all right to use the dimensionless units you determined previously. However, in any event write your final answer in terms of dimensional quantities, and interpret it using a random-walk argument.

(e) [10 min.] Consider now the rather strange limit in which the velocity field does not vary with time at all. Since the velocity also does not vary in space, we are saying that in each realization the velocity is a constant number. (Of course, that number varies in amplitude and sign from realization to realization.) Furthermore, let the Reynolds number $R$ be very large, although not infinity. Thus, we are now considering a limit of very strong turbulence. Consider the long-time, steady-state limit, in which the temperature field depends only on space. Using qualitative arguments (explain them), sketch a few representative realizations of the temperature field $T(x)$. (Consider both positive and negative velocity, and be sure that your pictures satisfy the boundary conditions.) Use this insight to qualitatively average all possibly temperature realizations in order to sketch what the mean temperature profile $(T(x))$ should look like in this limit. In particular, what is the mean profile when $R \to \infty$? Contrast these pictures with your previous steady-state solution for the quasilinear profile, and explain the differences.

(f) [5 min.] Determine the scaling of the steady-state flux in this limit of strong turbulence. You may use dimensional analysis or, possibly, a random-walk argument; you do not need to compute any numerical coefficient. However, if you invoke random walk, explain whether a random-walk argument should be expected to apply in this limit.
SECTION V - EXPERIMENTAL

CHOOSE EITHER PROBLEM 1 OR PROBLEM 2. THEN COMPLETE BOTH PROBLEMS 3 AND 4.

DON'T FORGET TO CORRECTLY NUMBER YOUR PROBLEMS WITH SECTION #, PROBLEM #.
1. The three graphs below show data taken by electron temperature measuring diagnostics, namely 90° Thomson scattering, a single Langmuir probe, and a SiLi x-ray detector. Evaluate the electron temperatures from the data.

**Langmuir Probe**

**Thomson Scattering**

Only evaluate $T_e$ for curve (A) in this figure.
SECTION V - PROBLEM 2 (DO NOT DO THIS PROBLEM IF YOU DID #1)

Plates in Plasma - 30 mins.

Two metal plates of weight \( W \) and surface areas \( S \) with a battery in between is suspended in a plasma by a weightless, insulating bar of length \( L \). The plasma properties are mass \( M \), density, \( n \), electron temperature, \( T_e(\neq 0) \) and \( T_i \approx 0 \). To express it another way, one metal plate is biased by an electrical potential, \( V \), with respect to the other. Ignore the size of the metal plate with respect to \( L \). Ignore the edge effect. Ignore the momentum imparted to the plates by electrons. Assume that ions and electrons recombine on the surface and remain there. Note that the double probe current, \( I \), is given as
\[
I = S J_0 \tan h \left( \frac{1}{2} \frac{V}{kT_e} \right), \quad \text{where} \quad J_0 = ne/kT_e/M
\]

1) Does the bar deviate from the vertical direction? Which way does the bar make the angle? Assume that the positively biased metal plate faces the + \( \theta \) direction.

2) Express the angle in terms of \( W \), \( n \), \( T_e \), \( L \), \( S \) and \( M \).

3) Could this principle be used to prevent the slowdown of a satellite in a space plasma?
SECTION V - PROBLEM 3

Artsimovitch Scaling - 15 mins.

In the ohmic discharges of the early small tokamaks, it was found that neoclassical "plateau" heat transport could explain the observed ion temperatures, $T_+$. Rederive the Artsimovitch formula

$$T_+ = 6 \cdot 10^{-10} \left( \frac{I B_T}{A G} \frac{R^2}{cm^2 cm^{-3}} \right)^{1/3}$$

where $I$ is the plasma current, $B_T$ the toroidal magnetic field, $R$ the major radius and $n_e$ the plasma density.

HINT: Artsimovitch used the fact that the quantity

$$\frac{T_e}{T_+} - 1 \frac{T}{(\frac{T_e}{T_+})^{3/2}} = .33$$

shows only a variation < 25% in the range $1.6 < \frac{T_e}{T_+} < 10$. Here $T_e$ is the electron temperature.
SECTION V - PROBLEM 4
Divertor - 15 mins.

1. a) What defines a divertor? (4)
   b) What are the functions of a divertor in a (hot) fusion reactor? (5)
   c) Describe, by means of labeled sketches, the following divertor configurations:
      (6)
      i) toroidal divertor
      ii) poloidal divertor
      iii) bundle divertor
SECTION VI - PROBLEM 1

Guiding Center Motions - 20 mins.

Consider a charged particle drifting in the fields

\[ \vec{B} = zB_0 e^{\alpha x} \]

\[ \vec{E} = yE_0 \]

where \( \alpha, E_0 \) and \( B_0 \) are constants.

Find the guiding-center trajectory and evaluate the particle energy as a function of time.
SECTION VI - PROBLEM 2

Plasma Radiation - 25 miss.

If a plasma were in true thermodynamic equilibrium, it would radiate like a blackbody, and that radiation is huge. For example, a tokamak would radiate about \( P_{BB} = 10^{23} T_{10}^4 a_1^{-1} \) W/cm\(^2\), where \( T_{10} \) is the plasma temperature in 10 keV and \( a_1 \) is the minor radius in meters.

(2 pts.) a) Luckily, a tokamak reactor is far from thermal equilibrium, but there are radiation losses, \( P_L \), due to synchrotron and bremsstrahlung emissions. How small a fraction of the blackbody power (\( P_L/P_{BB} \)) do you imagine is tolerable in these radiation losses in an economic tokamak reactor? Just give your answer as an order of magnitude.

(15 pts.) b) Suppose that, for a given reactor design, \( P_L/P_{BB} \) were independent of frequency and twice the tolerable level. What efficiency would one require in mirrors mounted around the tokamak, if the intention is to reduce the radiation losses to a tolerable level? (The efficiency of a mirror is \( \Gamma \) if it reflects a fraction \( \Gamma \) of incident radiation.)

(8 pts.) c) Is the mirror efficiency you required above realistic in terms of the reflectivity of common materials? What more accurate model of emissions drastically changes your requirements on the mirror reflectivity? Discuss your reasoning.