

DEPARTMENT OF ASTROPHYSICAL SCIENCES
PROGRAM IN PLASMA PHYSICS

GENERAL EXAMINATION, PART I

MAY 18, 1995

9 a.m. - 1 p.m.

- Answer all problems. Problem 4 has a choice of A or B (answer one only).
- The exam has been designed to require about 3 hours of work; however we have allowed you an extra hour. Thus the total time allotted for this day is 4 hours. Scores on questions will be weighted in proportion to their allotted time.
- Start each numbered problem in a new test booklet. Put your name, question # and part # on every booklet title page.
- When you do not have time to put answers into forms that satisfy you, indicate specifically how you would proceed if more time were available. If you do not attempt a particular problem, write on a sheet of paper "I have not attempted Problem ____ " and sign your name.
- All work on this examination must be independent. No assistance from other persons is permitted.
- No aids (books, calculators, etc.) except for an NRL formulary are permitted.

Problems for Part I, May 18, 1995

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|----|---|-------------------------|
| 1) | MHD Quickie | 15 points |
| 2) | Spectroscopy Quickie | 10 points |
| 3) | Alfven Wave | 40 45 points |
| 4) | Experimental - Do problem A OR B | 45 points |
| | 4a) Experimental Plasma Physics | |
| | 4b) Experimental Glow Discharge | |
| 5) | Applied Math | 40 points |
| 6) | Nonlinear Theory | 25 points |
| 7) | Mirror Quickie | 15 points |

Total - 190 points

Part I, Question 1

MHD Quickie (15 minutes)

The rotational transform in a stellarator is provided by the twisting of external field coils, and consequently the flux surfaces have complicated shapes which are not toroidally symmetric. Nevertheless, plasma pressure is still constant on each flux surface (in the ideal MHD approximation for a stationary equilibrium).

What is a flux surface? (5 points) Show why pressure is constant on a flux surface (in the ideal MHD approximation for a stationary equilibrium). (5 points) Give an example of a plasma device that operates with pressure not constant on flux surfaces. (5 points)

Experimental quickie (10 minutes)

Consider the observation of atoms or ions in a magnetized plasma by visible-light spectroscopy.

- a) Name, and briefly describe, four effects that can cause a shift or broadening of the emitted spectral lines.
- b) Estimate the thermal width of the resonance line ($\lambda = 5535 \text{ \AA}$, for the purists) emitted from barium neutrals at a temperature of 2 eV in a tenuous plasma with an electron temperature of 100 eV. What type of spectrometer could be used to perform such a measurement?

Damping of an Alfvén wave in a Partially Ionized Plasma

45 40 minutes

Find the damping rate of a small amplitude parallel propagating linearly polarized Alfvén wave propagating in a hydrogenic uniformly magnetized plasma that is 50 percent ionized. (The neutral and ion densities are equal.)

a) Take densities as $\rho_i = \rho_0$. You may assume that the densities are unperturbed in the wave. Take the equilibrium magnetic field to be $B_0 \hat{z}$ where B_0 is constant. Take the perturbed neutral and ion velocities v_i and v_n of the ions and neutrals respectively to be in the x direction. Take the wave quantities proportional to $\exp i(-\omega t + kz)$.

b) Consider the plasma to be described by magnetohydrodynamic equations with an additional force per unit volume of $-\rho_i \nu_{in}(\mathbf{v}_i - \mathbf{v}_n)$. Consider that the only force per unit volume on the neutrals is $-\rho_n \nu_{ni}(\mathbf{v}_n - \mathbf{v}_i)$, with $\nu_{in} = \nu_{ni}$ where these are the ion-neutral and neutral-ion collision frequencies respectively.

1. (15 minutes) Write down the correct linearized equations for v_{ix} and v_{nx} . Show that the $\mathbf{j} \times \mathbf{B}$ force can be written as $-\rho_i k^2 V_A^2 \mathbf{v}_i / (-i\omega)$ where $V_A^2 = B^2 / \sqrt{4\pi\rho_i}$.

2. (10 minutes) Derive an equation for complex ω as a function of wave number k . (Don't solve it yet.)

3. (10 minutes) Solve for the real part of the frequency to lowest order in two limits: $\omega \gg \nu_{in}$ and $\omega \ll \nu_{in}$. What is the effective density for the Alfvén wave in these two limits. (Can you give a physical justification for these values?)

4. (10 minutes) Solve for the damping rate (the imaginary part of ω) to lowest order in the same two limits. (Can you give a physical argument for the result in the high frequency limit?)

Do A or B only

Day I, Question 4A

Experimental plasma physics (45 minutes)

Startup in tokamaks begins with a Townsend avalanche in a poloidal field null. This puts certain requirements on the quality of the field null, the toroidal loop voltage, and the gas pressure. Discuss these requirements in terms of Townsend's first coefficient for deuterium, $\alpha \cong A p \exp(-Bp/E)$, where $A = 510 \text{ m}^{-1}\text{torr}^{-1}$, and $B = 12,500 \text{ V m}^{-1}\text{torr}^{-1}$.

Do A or B only
Day I, Question 4B

Experimental methods: The Glow Discharge (45 minutes)

a) Apparatus

Describe the apparatus needed to make an anomalous (DC) glow discharge. Include equipment for: 1) obtaining (and measuring) the appropriate gas pressure; 2) providing (and measuring) the requisite electrical power; and 3) measuring the plasma electron temperature and space potential. (15 minutes)

b) Breakdown phenomena (10 minutes)

i) Describe (sketch) the relation between breakdown voltage V_B and pressure and explain the physical processes responsible for its shape.

ii) Rank the following gases in order of lowest to highest V_B : Ar, Ne, Air

c) Describe the (visible light and space potential) architecture of the glow discharge. Pay special attention to the structure in the sheath. (10 minutes)

d) Explain why a hollow cathode configuration allows higher densities to be achieved than a planar configuration. (10 minutes)

Part I, Question 5

Applied Math

1. (40 minutes) Consider the equation

$$\epsilon \frac{d^2 y}{dx^2} + e^x \frac{dy}{dx} - y = 0,$$

on the interval $-1 \leq x \leq 1$, with the boundary conditions $y(-1) = 1$, $y(1) = .01$. Find the leading order asymptotic approximation to the solution for y in the limit $\epsilon \rightarrow 0_+$. Give a single expression that is uniformly valid in the interval $-1 \leq x \leq 1$. Sketch what the solution looks like.

Nonlinear Theory

(25 points)

A bump-on-tail instability can be modeled by the equations

$$dx/dt = v$$

$$dv/dt = A \sin(x-t)$$

with x a particle coordinate, and $A \sin(x-t)$ an electric potential.

a) Calculate the maximum velocity excursion from the unperturbed ($A=0$) motion. Show the particle trajectories in the x - v plane.

b) Find the trapping frequency ω_t for an orbit deeply trapped in the wave.

c) The initial particle distribution is

$$F = \begin{cases} v/2 & 0 < v < 2 \\ 0 & v > 2 \end{cases}$$

Sketch the particle distribution for $\omega_t t \gg 1$. Calculate the fractional energy change in the particle distribution.

d) If the mode has a linear growth rate γ , what will be the approximate saturation mode amplitude A ? Why?

Part I, Question 7

2. Mirror Quickie (15 points)

- (a) (5 pts) Describe how mirror confinement works, and sketch single particle orbits.
- (b) (5 pts) Derive a trapping condition for mirror confined particles in terms of the particle's initial coordinates and the mirror ratio.
- (c) (5 pts) Please explain briefly what fluid instability might be exhibited by a plasma confined in a simple mirror?

DEPARTMENT OF ASTROPHYSICAL SCIENCES
PROGRAM IN PLASMA PHYSICS

GENERAL EXAMINATION, PART II

MAY 19, 1995

9 a.m. - 1 p.m.

- Answer all problems. Problem 2 and 6 have a choice of A or B (answer one in each question only).
- The exam has been designed to require about 3 hours of work; however we have allowed you an extra hour. Thus the total time allotted for this day is 4 hours. Scores on questions will be weighted in proportion to their allotted time.
- Start each numbered problem in a new test booklet. Put your name, question # and part # on every booklet title page.
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Problems for Part II, May 19, 1995

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|-----|---|-----------|
| 8) | Tokamak Orbits | 20 points |
| 9) | General - Do problem A OR B | 45 points |
| | 9a) Waves | |
| | 9b) Transport | |
| 10) | Kinetic Theory | 55 points |
| 11) | Kinetic Quickie | 15 points |
| 12) | Experimental | 30 points |
| 13) | Waves Quickie | 10 points |
| 14) | Applied Math/Computational - Do problem A OR B | 20 points |
| | 14a) Applied Math | |
| | 14b) Computational | |

Total - 195 points

Part II, Question 8

1. Tokamak Orbits (20 points)

- (a) (5 pts) What are the constants of the motion for particle orbits in a tokamak?
- (b) (10 pts) Sketch the orbits of copassing, passing, and banana trapped ions in a tokamak.
- (c) (5 pts) Write an approximate expression for the banana width of a trapped particle in terms of its initial parallel velocity and the poloidal magnetic field. Explain briefly how you arrive at this expression.

Waves Problem (40 points)

Consider the kinetic electrostatic wave dispersion relation in a uniform single ion species Maxwellian magnetized plasma. In the ion cyclotron range of frequency with sufficient density, one can approximate as $\epsilon_{xx} \approx \chi_{xx}^i$ and $\epsilon_{zz} \approx \chi_{zz}^e$ where

$$\chi_{xx}^i = \frac{\omega_{pi}^2}{k_{\parallel} \omega V_{Ti}} \sum_{n=-\infty}^{\infty} e^{-\lambda} \frac{n^2 I_n}{\lambda} Z_0(\zeta_n)$$

$$\text{and } \zeta_n = \frac{\omega - n\Omega_i}{k_{\parallel} V_{Ti}}, \quad \lambda = \frac{k_{\perp}^2 \kappa T_i}{m \Omega_i^2}, \quad Z_0(\varsigma) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dz \frac{e^{-z^2}}{z - \varsigma}, \quad \text{Im } \varsigma > 0$$

(a) For hot electrons and large ζ_n for ions, indicate qualitatively the regions in frequency space of possible wave propagation for $0 < \omega < 4\Omega_i$ [Do not neglect the finite-Larmor-radius (FLR) terms here.] (10 points)

Note: If you do not know the exact forms of I_n , make your best guess.

(b) Derive the wave dispersion relation neglecting the FLR terms and damping, and assuming electrons are hot. Sketch it for $0 < \omega < 4\Omega_i$. Can you name the wave? (10 points)

(c) Using the dispersion relation obtained in (b) for the real part but including the kinetic term for the imaginary part, obtain an expression for the damping (assuming weak damping) for $\omega \approx 4\Omega_i$. Plot the damping as a function of the wave frequency (qualitatively) for $\omega \approx 4\Omega_i$. (20 points)

Transport Problem:

for General Exams

~ (45 minutes)

This problem deals with formulating the analysis needed to determine the influence of impurities on anomalous transport. Specifically, analyze the effect of carbon impurities on the well known ion temperature gradient instability. (Your task is to set up the problem, but not to actually find the growth rates.)

- [25] (a) Obtain the simplest radially local linear density responses to electrostatic perturbations by electrons, hydrogen ions, and cold carbon impurities in a slab geometry. Consider

$$(v_{Th})_c \ll (v_{Th})_H < \frac{\omega}{k_{\parallel}} < (v_{Th})_e$$

with $(v_{Th})_j$ being the thermal velocity of species j and ω/k_{\parallel} being the parallel phase velocity of the waves. Finite gyroradius effects can be ignored. [Hints: You can start with either the drift-kinetic equation or with fluid equations. Terms of order $(k_{\parallel}(v_{Th})_H/\omega)^2$ must be kept in the hydrogen response. A formula you might need:
 $\int dv_{\parallel} f_M v_{\parallel}^4 = 3 n_o (T/m)^2.$]

- [10] (b) Obtain the quasineutrality condition from Poisson's equation and apply it to this problem to obtain the local dispersion relation. (Do not bother solving the dispersion relation for ω .)
- [5] (c) What are the charge neutrality constraints which must be taken into account in this analysis?
- [5] (d) How does one proceed to formulate the radial eigenmode analysis in a sheared slab geometry from the local result of part (b)?

Kinetic Theory (55 mins)

Consider a uniform electron-ion plasma with stationary infinitely massive ions to which a steady-state constant electric field $E_0 \hat{z}$ is applied. Assume that the plasma has reached a steady state and that collisions may be described by the Landau collision operator.

(a) [15 mins] Write the kinetic equation for the electrons including electron-electron and electron-ion collisions. Keep the electron-electron collisions term in symbolic terms, but derive an explicit form for the electron-ion collision term.

(b) [5 mins] Linearize the kinetic equation under the assumption that the electric field is weak and that the electron distribution is close to a stationary Maxwellian.

(c) [15 mins] Take the z -directed momentum moment of the linearized kinetic equation, and thus relate the electron distribution function to the electric field.

(d) [15 mins] Assuming that the electron distribution is a drifting Maxwellian, use the result in part (c) to compute the drift and hence derive an expression for the electrical conductivity.

(e) [5 mins] In fact, the electron distribution is *not* a drifting Maxwellian. Is the result you derived in part (d) an overestimate or an underestimate of the electrical conductivity? Why?

Hint: The Landau form of the collision operator is $(\partial f_a / \partial t)_{\text{coll}} = - \sum_b (\partial / \partial \mathbf{v}) \cdot \mathbf{J}^{a/b}$, with

$$\mathbf{J}^{a/b} = \frac{n_b e_a^2 e_b^2 \log \Lambda}{8\pi \epsilon_0^2 m_a} \int d^3 \mathbf{v}' \frac{u^2 |\mathbf{u}| - \mathbf{u} \mathbf{u}}{u^3} \cdot \left\{ \frac{f_a(\mathbf{v})}{m_b} \frac{\partial f_b(\mathbf{v}')}{\partial \mathbf{v}'} - \frac{f_b(\mathbf{v}')}{m_a} \frac{\partial f_a(\mathbf{v})}{\partial \mathbf{v}} \right\},$$

where $\mathbf{u} = \mathbf{v}' - \mathbf{v}$.

Part II, Question 11

Kinetic quickie (15 mins)

The Braginskii equations, describing the classical transport of plasma in a magnetic field, are an asymptotic limit of the full kinetic equations. State the asymptotic ordering used.

What modifications to this ordering are needed to obtain the "neoclassical" transport equations? What new physics is included in the neoclassical equations which is absent from the classical equations.

Experimental plasma physics (30 minutes)

Discuss the principles of charge-exchange recombination spectroscopy. What can it measure, what are the key elements of the atomic physics mechanism, what are some of the key advantages of this technique and key limitations?

Part II, Question 13

Waves Quickie

1. (10 points) Consider a neutral plasma with equal quantities of regular hydrogen ions, H_1^+ , and negative hydrogen ions, H_1^- (i.e., no electrons). Assume both ion species are fully equilibrated with each other, i.e., $T^+ = T^-$. What type of electrostatic wave could this plasma support? Explain.

Do A or B only

Part II, Question 14A

2. (20 minutes) Find the leading behavior of

$$\int_1^{\infty} e^{-\alpha(t+1/t)} \ln(2+t) dt$$

as $\alpha \rightarrow +\infty$.

Iterative Finite Difference Method for Solving Elliptic Equations:

Consider the 2D Poisson's equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = R(x,y)$$

The boundary values and the function R are assumed given.

Suppose we wish to solve this on a 2D rectangular region of size $L_x \times L_y$, using N_x grid points in the x direction and N_y in the y direction. Write down an iterative finite difference method for solving this equation, and derive an estimate for how many iterations would be required to reduce the error by $1/e$. What is a faster method for solving this equation?