DEPARTMENT OF ASTROPHYSICAL SCIENCES
PROGRAM IN PLASMA PHYSICS

GENERAL EXAMINATION, PART I

MAY 16, 2005

9 a.m. - 1 p.m.

- Answer all problems.

- The exam has been designed to require about 3 hours of work; however we have allowed you an extra hour. Thus the total time allotted for this day is 4 hours. Scores on questions will be weighted in proportion to their allotted time.

- Start each numbered problem in a new test booklet. Put your name, question # and part # on every booklet title page.

- When you do not have time to put answers into forms that satisfy you, indicate specifically how you would proceed if more time were available. If you do not attempt a particular problem, write on a sheet of paper "I have not attempted Problem ____ " and sign your name.

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Problems for Part I, May 16, 2005

1)	Asympototic methods	40 points
2)	Experimental methods	25 points
3)	General phenomena	20 points
4)	Neoclassical theory	45 points
5)	Irreversible Processes	50 points
		Total - 180 points

Part I, Question 1

Asympototic Methods

Math Problem - (40 points)

Consider the differential equation

$$\frac{d^2y}{dx^2} + \frac{1}{x^2}\frac{dy}{dx} + \frac{1}{x^2}y = 0.$$

A. Find the leading asymptotic form of both real solutions for $x \simeq 0$.

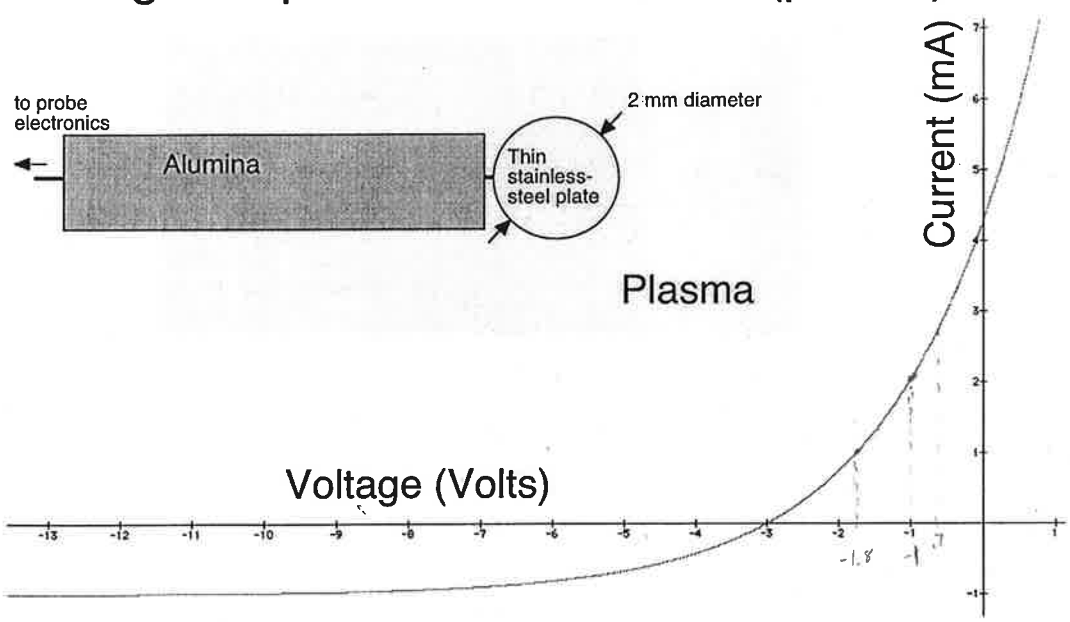
B. Find the leading asymptotic form of both real solutions for $x \simeq \infty$.

Experimental quickie (25 pts)

Consider the Langmuir probe and its characteristic shown below. The probe is immersed in an infinite, uniform, unmagnetized Argon plasma having Maxwellian distributions of "warm" electrons and much cooler ions. Neutral atoms are of low enough density to be unimportant.

(2 pts) **a)** What is the value of the floating potential?
(5 pts) **b)** Estimate the electron temperature to an accuracy better than 10%.
(2 pts) **c)** What is the likely charge state of the argon ions?
(8 pts) **d)** To an accuracy of 10%, what is the argon ion density many Debye lengths from the probe?
(3 pts) **e)** At what value of current do you expect the electron saturation current to be?
(5 pts) **f)** Describe how the I-V characteristic would change if a low density ($n_b/n_e \sim 10^{-3}$) mono-energetic (E = 10 eV) uni-directional (normal to plate) electron beam also existed in the plasma.

Langmuir probe characteristic (partial)



Part I, Question 3

General Phenomena

1. [10 points] MHD ordering.

Starting with the fluid equation for the electron momentum, and ignoring the electron inertia, one gets the generalized Ohm's law (in cgs units):

$$\vec{E} + \frac{\vec{u} \times B}{c} = \eta\vec{j} + \frac{\vec{j} \times \vec{B}}{en_e c} - \frac{\nabla p_e}{en_e}$$

The final two terms are usually ignored in MHD. Pick one of these two terms and show, by means of an order-of-magnitude estimate, in what limit that term may be neglected.

2. [10 points] Transport processes Provide a random walk argument for electron and ion thermal conductivities χ_e and χ_i along magnetic field lines, explaining the dependence on the collision frequency. Give an estimate of the ratio χ_i/χ_e (assuming $T_e \sim T_i$ and $Z = 1$ for simplicity.

Part 1, Question 4

Neoclassical Theory
45 points

A toroidal plasma with a pressure gradient in the radial direction and a parallel electric field, $E_\parallel$, will relax via Coulomb collisions. In this problem, consider the associated "neoclassical" transport in the long mean-free-path "banana regime," and ignore, for simplicity, any temperature gradients.

(A) 25 minutes

What is the trapped-particle flux in terms of these driving forces? Specifically, provide a simplified (heuristic) derivation of the coefficients for the pressure gradient ($-T_e dn_0/dr$) and $-n_0 E_\parallel$ driving forces. Use the conservation of canonical angular momentum to estimate the inward radial velocity of the trapped particles.

(B) 10 minutes

How is the classical (Spitzer) conductivity, $\sigma_\parallel$, modified in this regime?

Give a simplified (heuristic) derivation for the neoclassical current density -- again specifying the coefficients for the driving forces.

(C) 10 minutes

What is Onsager Symmetry as applied to this problem? How can it be usefully applied in writing down the solutions in parts (A) and (B)?

Part I, Question 5

Irreversible Processes
[50 points]

- ***Read through the entire problem before you begin any calculations.***

- ***Given some basic understanding, the various parts of this question are independent of each other.***

Consider unmagnetized electrons interacting with discrete, infinitely massive ions. The electrons obey the usual kinetic equation

$$Df/Dt \equiv \partial_t f + \boldsymbol{v} \cdot \boldsymbol{\nabla} f + (q/m)\boldsymbol{E} \cdot \partial_{\boldsymbol{v}} f = -C[f], \tag{1}$$

where (the electrostatic) $\boldsymbol{E}$ is determined from Poisson's equation. Assume that you can use the Lorentz collision operator

$$C[f] = \nu(v) L^2 f, \tag{2}$$

where

$$\nu(v) = \frac{3\sqrt{2\pi}}{4} \frac{1}{\tau_e} \left(\frac{v_{te}^3}{v^3} \right) \tag{3}$$

(τ_e being Braginskii's collision time) and the eigenfunctions of L^2 are the spherical harmonics:

$$L^2 Y_l^m(\theta, \phi) = l(l+1) Y_l^m(\theta, \phi). \tag{4}$$

(The neglect of electron–electron collisions is correct for $Z \to \infty$.) **Your goal is to discuss the Chapman–Enskog analysis of this problem in as much detail as time allows, by addressing each of the following steps:**

(a) [5 points] Argue that the lowest-order distribution can be taken to be the unshifted Maxwellian

$$f_0(v) = \left(\frac{n(\boldsymbol{x}, t)}{\overline{n}} \right) [2\pi v_t^2(\boldsymbol{x}, t)]^{-3/2} \exp[-v^2/2v_t^2(\boldsymbol{x}, t)], \tag{5}$$

where

$$v_t^2(\boldsymbol{x}, t) = T(\boldsymbol{x}, t)/m. \tag{6}$$

(Problem continues on next page.)

(b) [5 points] The first-order equation can be written as

$$\frac{Df_0}{Dt_1} = -C[f_1]. \tag{7}$$

What are the solvability constraints on this equation? You may state those on physical grounds; you don't need to prove mathematical theorems.

(c) [25 points] In appropriately dimensionless units, the transport equations (for small perturbations from equilibrium) can be shown to be

$$\begin{aligned} \partial_t n &= -\alpha[2(1+\epsilon)n + 5\epsilon T], & \text{(8a)} \\ \partial_t T &= -\alpha\left(\frac{20}{3}(1+\epsilon)n + 22\epsilon T\right), & \text{(8b)} \end{aligned}$$

where

$$\alpha \stackrel{\text{def}}{=} \frac{16}{3\pi} \tag{9}$$

and $\epsilon \stackrel{\text{def}}{=} -\nabla^2 \to k^2$. **Show how to work with the first-order equation and the solvability constraints to derive the factor of** 5α (in the nT component of the transport matrix). If you don't have time to complete the arithmetic, say in detail how you would proceed. Be sure to write down at least some sort of specific equation that you need to solve.

Hint: There's no particular need to introduce a scalar-product notation if you don't want to; just solve the equation directly. If you do use scalar products, indicate very clearly what the operations mean.

(d) [8 points] Show that the hydrodynamic eigenvalue of the system (8) is

$$\lambda = \frac{2}{3}\left(\frac{128}{3\pi}\right)\epsilon \tag{10}$$

(and explain why it is a "hydrodynamic" eigenvalue).

(e) [7 points] According to Braginskii, the thermal conductivity coefficient κ_e in the heat-flow law $\boldsymbol{q}_e = -n_e\kappa_e\boldsymbol{\nabla}T_e$ is (for $Z = \infty$)

$$\kappa_e = \frac{128}{3\pi}, \tag{11}$$

which is obviously three-halves times the result (10). **Physically, would you have expected this result, or is this a coincidence?** Worry about the fact that Bragkinskii's temperature equation $\frac{3}{2}n dT/dt = -nT\boldsymbol{\nabla}\cdot\boldsymbol{u} - \boldsymbol{\nabla}\cdot\boldsymbol{q} + \cdots$ contains a flow term, while there is no flow variable in Eqs. (8).

(Problem continues on next page.)

Hints:

- In a spherical-polar coordinate system, $\cos\theta = Y_1^0(\theta, \phi)$.
- $\Gamma(z) = \int_0^\infty dt\, t^{z-1} e^{-t}$.
- $\Gamma(z+1) = z\Gamma(z)$.

DEPARTMENT OF ASTROPHYSICAL SCIENCES
PROGRAM IN PLASMA PHYSICS

GENERAL EXAMINATION, PART II

MAY 17, 2005

9 a.m. - 1 p.m.

- Answer all problems. Problem 3 has a choice of (A) or (B). Answer only one.

- The exam has been designed to require about 3 hours of work; however we have allowed you an extra hour. Thus the total time allotted for this day is 4 hours. Scores on questions will be weighted in proportion to their allotted time.

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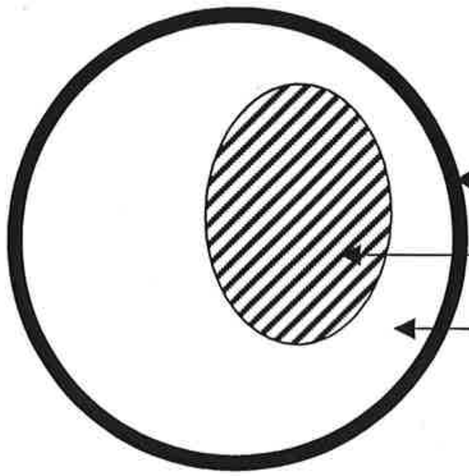
<u>Problems for Part II, May 17, 2005</u>

1)	MHD	40 points
2)	Kinetic quickies	15 points
3)	Do question A or B	10 points
	Waves quickie	
	OR	
	Irreversible quickie	
4)	Waves – long problem	45 points
5)	Thomson scattering	20 points
6)	Experimental probes	25 points
7)	Mirror orbits	25 points

<u>Total - 180 points</u>

Part II, Question 1

MHD, 40 points



We will consider the situation pictured above, with a plasma surrounded by a vacuum region, and a perfectly conducting wall surrounding both. Assume that the plasma is cold (zero pressure), and that the plasma-vacuum interface corresponds to a flux surface.

For the problems below, you may find the following identities useful:
Given a vector, **A**, and a scalar f,

$$\nabla\cdot(f\mathbf{A}) = f\nabla\cdot\mathbf{A} + \mathbf{A}\cdot\nabla f .$$

Given two vectors, **A** and **C**,

$$\nabla\cdot(\mathbf{A}\times\mathbf{C}) = \mathbf{C}\cdot\nabla\times\mathbf{A} - \mathbf{A}\cdot\nabla\times\mathbf{C} .$$

Given a scalar function $f(\mathbf{r},t)$, define $F(t) \equiv \int_V f(\mathbf{r},t)d^3x$, where V denotes the integration region. Then

$$\frac{dF}{dt} = \int_V \frac{\partial f}{\partial t} d^3x + \int_S f\hat{\mathbf{n}}\cdot\mathbf{u}d^2x ,$$

where S denotes the bounding surface of V, $\hat{\mathbf{n}}$ is the unit normal vector pointing outward, and **u** is the local velocity of the boundary.

a) The MHD energy density in the plasma is given by

$$w_p = \frac{1}{2}\rho v^2 + \frac{B^2}{2\mu_0},$$

where ρ is the mass density, v is the plasma velocity, and B is the magnitude of the magnetic field. Using the ideal MHD equations, obtain an expression for the time derivative of the total energy in the plasma, and express it in terms of an integral over the plasma boundary. (10 points)

b) The vacuum energy density is $w_v = B^2/(2\mu_0)$. Give an expression for the time derivative of the total energy in the vacuum region, expressed as an integral over the boundaries of the vacuum region. (5 points)

c) What is the time derivative of the total energy in the region surrounded by the conducting wall? State what jump conditions you are using at the plasma-vacuum interface, and what boundary conditions you are using at the conducting wall. (10 points)

d) Why is there no term of the form $\varepsilon_0 E^2/2$ in the MHD expression for w_P? Using the ideal MHD equations, estimate the relative magnitude of $\varepsilon_0 E^2/2$ and $B^2/(2\mu_0)$ in the plasma. (15 points)

Part II, Question 2
Kinetic Quickies
15 points total

This problem deals with basic drift motions:

(a) 5 points

From simple fluid pressure balance considerations (scalar pressure, no temperature gradient, and $B = B_0\hat{z}$) for species j, derive an expression for the diamagnetic drift velocity. Explain why this is called a "diamagnetic" velocity.

(b) 10 points

From a simple single particle orbit picture, first obtain the $\underline{E} \times \underline{B}$ drift velocity and then go to next order in ω / Ω (with Ω being the cyclotron frequency and ω being a measure of the time-rate-of-change of the $\underline{E}$-field), to obtain the polarization drift velocity. Consider a uniform magnetic field ($\underline{B}$).

Part II, Question 3(A)

High Harmonic Fast Waves in NSTX
10 points

One method for heating deuterium plasmas in NSTX involves launching high harmonic fast waves into an NSTX plasma at a frequency of 30 MHz. Assume that the plasma density in NSTX is 3 X 10^{13} cm^{-3} and the equilibrium magnetic field strength is 0.4 T in the central regions of the plasma. The minor radius of the plasma on the midplane is approximately 62 cm. Estimate the perpendicular wavelength of the HHFW in the cerntral region of the plasma (assuming that the parallel wavelength is small compared to the perpendicular wavelength and hence can be neglected in this estimate.) Comment on the applicability of ray tracing methods for simulating HHFW propagation and damping in NSTX.

Part II, Question 3(B)

Irreversible Processes
[10 points]

According to Braginskii, the classical friction force $\boldsymbol{R}$ in a strongly magnetized plasma looks like

$$\boldsymbol{R} \propto \alpha \boldsymbol{u}_{\|} + \boldsymbol{u}_{\perp}, \tag{1}$$

where $\boldsymbol{u} \stackrel{\text{def}}{=} \boldsymbol{u}_e - \boldsymbol{u}_i$ is the relative fluid flow and α is a certain numerical coefficient. For atomic number $Z = 1$, various methods of calculation lead to different results for α, as follows:

$$\text{Method 1: } \alpha = 1 \tag{2a}$$

$$\text{Method 2: } \alpha \approx 0.5 \tag{2b}$$

$$\text{Method 3: } \alpha = 3\pi/32. \tag{2c}$$

Identify the physical approximation involved with each method. Which one is exact? Give arguments that explain the relative sizes of the other methods.

Part II, Question 4
Parallel Wave Propagation in a Hot, Uniform Plasma
[45 points total]

Consider a hot, uniform, magnetized plasma in which both the electrons and ions are isotropic, non-drifting Maxwellians. Assume that the equilibrium magnetic field, B_0, points in the z-direction and that the waves of interest are propagating purely along the equilibrium magnetic field. In this limit, the hot plasma susceptibility for species, s, can be written in the form:

$$\overleftrightarrow{\chi} = \begin{bmatrix} \chi_\perp & -i\chi_x & 0 \\ i\chi_x & \chi_\perp & 0 \\ 0 & 0 & \chi_{zz} \end{bmatrix} \quad \text{where}$$

$$\chi_\perp = \frac{1}{2}\frac{\omega_p^2}{\omega^2}\frac{\omega}{k_{//} v_t}\left[Z(\zeta_{-1}) + Z(\zeta_1)\right]$$

$$\chi_x = \frac{1}{2}\frac{\omega_p^2}{\omega^2}\frac{\omega}{k_{//} v_t}\left[Z(\zeta_{-1}) - Z(\zeta_1)\right]$$

$$\chi_{zz} = \frac{2\omega_p^2}{k_{//}^2 v_t^2}\left\{1 + \frac{\omega}{k_{//} v_t} Z(\zeta_0)\right\}$$

$$\varsigma_n = \frac{\omega - n\Omega}{k_{//} v_t}$$

(a) **[15 points]** In terms of the susceptibility tensor elements, write down the dispersion relations for the three independent waves that can propagate in this plasma.

(b) **[5 points]** For a quasineutral plasma consisting of electrons and hydrogen, which of these three waves can experience a resonance at the fundamental ion cyclotron frequency.

(c) [25 points] Derive an approximate dispersion relation for the wave in part (b) that is valid for ω real and very close to Ω_H. Comment on the magnitude of $n_{//}$ for $\omega \sim \Omega_H$ in a hot plasma.

Useful information:

$$Z(\xi >> 1) \sim i\,\sigma\sqrt{\pi}\, e^{-\xi^2} - \frac{1}{\xi}\left[1 + \frac{1}{2\xi^2} + \ldots\ldots\right] \quad where \quad \sigma = \begin{cases} 0 & if \quad y > |x|^{-1} \\ 1 & if \quad y < |x|^{-1} \\ 2 & if \quad y < -|x|^{-1} \end{cases}$$

$$Z(\xi << 1) \sim i\sqrt{\pi}\, e^{-\xi^2} - 2\xi\left[1 - \frac{2\xi^2}{3} + \ldots\ldots\right]$$

Part II, Question 5
Thomson Scattering
20 points

(a) Derive the Thomson scattering cross-section.

(b) Calculate the fraction of photons incoherently scattered from a 1 cm path length of laser beam from a plasma with an electron density of 1e20 m^{-3} with a solid angle of detection of 0.01 sr. Assume 90 degrees scattering.

Helpful numbers: radius of an electron $r_e = 2.8 \times 10^{-15}$ m.

Part II, Question 6
Long Experimental Question
Langmuir Probes
25 points

In the absence of secondary electron emission, the condition that a probe or other particle collecting surface immersed in a plasma be at the floating potential (i.e. conduct no current to ground) is satisfied when the electron and ion currents are equal, or $j_i = j_e$:

$$n_e\left(\frac{kT_e}{2\pi m_e}\right)^{1/2}\exp\left(\frac{-e(V_{sp}-V_f)}{kT_e}\right)=0.6n_e\left(\frac{kT_e}{m_i}\right)^{1/2}$$

a) What modification to the above relationship is introduced by a nonzero secondary electron emission coefficient?
b) What is the effect (qualitative) of a moderate secondary electron emission coefficient on the floating potential? Consider δ_e=0.5 as a quantitative example.
c) For what secondary electron emission coefficient would the particle collecting surface (or probe) float at the space potential? If it develops that no material on earth has such a secondary electron emission coefficient, can you think of another approach which would allow a probe of some type to float at the space potential?
d) If the particle collecting surface were a conducting limiter to which a voltage bias could be applied, what effect would biasing the limiter at the space potential have on the power flow to the limiter?
e) If the conducting limiter were replaced with an insulating limiter (obviously not biased in this case), would there be an effect on the power flow to the limiter? Compare to an unbiased limiter, and to the situation in (d) above.

1. Mirror Orbits (25 points)

(a) (5 pts) Describe briefly how mirror confinement works, and sketch single particle orbits. Be sure to note all drift motions.

(b) (10 pts) Derive a trapping condition for mirror confined particles in terms of the particle's initial coordinates and the mirror ratio.

(c) (5 pts) Suppose that the plasma trapped in the mirror field is non-neutral, so that the plasma has a potential $\phi(z) = \phi_0 z^2/L^2$, where z is the axial position. Suppose that the magnetic field is of the form $B(z) = B_0(1 + z^2/L^2)$. Calculate the turning point in terms of initial coordinates and B_0.

(d) (5 pts) For potential $\phi_0 \neq 0$, plot the regimes of trapped and untrapped ions and electrons at the position $z = 0$ in terms of initial perpendicular and parallel velocities at $z = 0$. Consider a particle trapped if it has turning point before some potion $z = z_{max}$. Consider separately the effect of positive or negative potential.