DEPARTMENT OF ASTROPHYSICAL SCIENCES, PROGRAM IN PLASMA PHYSICS

GENERAL EXAMINATION, PART I

May 2, 2022

9:00 a.m. – 1:40 p.m.

- Today's exam (Part I) contains 6 problems on pages 2–9. Attempt all problems.
- Today's exam has been designed to require three hours of work (180 points). However, you are allowed one extra hour for problem solving, 20 extra minutes for "mask break", and 20 extra minutes for uploading your answers, so the total time allotted for today is 4 hours 40 minutes. The scores on the questions will be weighted in proportion to their allotted time.
- Start each numbered problem on a new page. Put your name and the question number on each page.
- When you do not have time to put answers into forms that satisfy you, indicate ***specifically*** how you would proceed if more time were available. If you do not attempt a particular problem, write on the booklet "I have not attempted Problem ___" and sign your name.
- All work on this examination must be independent. No assistance from other persons is permitted.
- An NRL formulary is permitted. No other aids (books, calculators, etc.) are allowed.

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I.1: General Plasma Physics [50 points]

Consider electrons in a magnetic mirror machine with mirror ratio R and mirror axis in the $\hat{z}$ direction. The electrons are sufficiently dilute that collisions are completely ignorable. Similarly, any self-generated electric or magnetic fields within the plasma are completely ignorable.

(a) [**6 points**] Derive the trapping condition in midplane ($z = 0$) energy coordinates, $W_{\perp 0}$ and $W_{\| 0}$, respectively the perpendicular and parallel energies as the midplane is crossed. Sketch in $W_{\perp 0}$-$W_{\| 0}$ space the region of trapped electrons.

(b) [**6 points**] Suppose that the magnetic field near the axis can be approximated as

$$\boldsymbol{B} = \begin{cases} B_0(1 + z^2/L^2)\hat{z}, & \text{if } |z| < cL \\ B_0(1 + c^2)\hat{z}, & \text{if } |z| > cL, \end{cases}$$

where c is a dimensionless constant, L is a constant with dimensions of length and B_0 is a constant with dimensions of magnetic field. Show that the turning points for trapped electrons obey: $z_T^2/L^2 = W_{\| 0}/W_{\perp 0}$. Note that one can write $R = 1 + c^2$.

(c) [**8 points**] Now suppose that $B_0 = B_0(t)$ is a very slowly varying function of time, such that it is monotonically increasing in going from $B_0(t = 0) = B_{0i}$ to $B_0(t = t_f) = B_{0f}$, where $B_{0f}/B_{0i} = \beta$. Calculate the changes in the perpendicular and parallel midplane energies. Note: by slowly varying we mean slow enough that the parallel particle motion can be considered to be adiabatic.

(d) [**10 points**] Suppose that a measure of being "more trapped" is that the change in the ratio $S \equiv W_{\perp 0}/W_{\| 0}$ is greater than zero. Show that all trapped electrons become "more trapped" for $\beta > 1$. Sketch in $W_{\perp 0}$-$W_{\| 0}$ space showing how the region of initially marginally trapped electrons (the trapped-passing boundary) becomes more trapped.

(e) [**10 points**] Suppose now that instead of $B_0 = B_0(t)$ being a very, very slowly varying function of time, it changes on a time scale very fast compared to the bounce time of trapped electrons but very slowly compared to a gyro-period, while increasing in going from $B_0(t = 0) = B_{0i}$ to $B_0(t = t_f) = B_{0f}$, where $B_{0f}/B_{0i} = \beta$. Suppose that the sudden increase occurs for trapped electrons at position z. Define the local mirror ratio $R_z \equiv B(z)/B_0$. Show that the change the midplane coordinates is then:

$$\Delta W_{\perp 0} = (\beta - 1)W_{\perp 0i}, \qquad \Delta W_{\| 0} = (R_z - 1)\Delta W_{\perp 0} \tag{1}$$

where $(W_{\| 0i}, W_{\perp 0i})$ and $(W_{\| 0f}, W_{\perp 0f})$ are the initial and final midplane energies, respectively, and $\Delta W_{\perp 0} = W_{\perp 0f} - W_{\perp 0i}$ and $\Delta W_{\| 0} = W_{\| 0f} - W_{\| 0i}$ are the changes in the energies.

(f) [**10 points**] Show that for this case all electrons are "no less trapped", or, in other words $\Delta S \geq 0$. For which electrons is the equality realized? *Hint:* It may help to write this condition as:

$$\Delta S = \frac{W_{\perp 0i}\Delta W_{\perp 0}}{W_{\parallel 0i}W_{\parallel 0f}}\left[\frac{W_{\parallel 0i}}{W_{\perp 0i}} - (R_z - 1)\right] \geq 0.$$

I.2: MHD Short Problem [15 points]

Derive the Grad-Shafranov equation for a plasma in slab geometry with the magnetic field independent of z where (x, y, z) are Cartesian coordinates. Which quantities are functions only of Ψ? Show this. What quantities need to be specified in order to solve this equation?

I.3: Waves and Instabilities [45 points]

Consider waves in nonmagnetized electron plasma with average density n_0 and stationary ions. Suppose that the electron distribution f is governed by the collisional Vlasov equation

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{x}} + \frac{e}{m}\left(\tilde{\boldsymbol{E}} + \frac{1}{c}\boldsymbol{v} \times \tilde{\boldsymbol{B}}\right) \cdot \frac{\partial f}{\partial \boldsymbol{v}} = -\nu(f - f_0), \tag{1}$$

where $\nu = \text{const}$, $f_0 = f_0(\boldsymbol{v})$ is a homogeneous equilibrium distribution with zero average velocity, tildes denote wave fields, $e < 0$, and the remaining notation is as usual.

(a) [4 points] Assume $\tilde{\boldsymbol{E}} = \mathrm{Re}[\boldsymbol{\mathcal{E}}(t)\mathrm{e}^{\mathrm{i}\theta(t,\boldsymbol{x})}]$, where $\boldsymbol{\mathcal{E}}$ is a slow complex envelope and θ is a rapid real phase. Assume that the wave is linearly polarized, i.e., $\boldsymbol{\mathcal{E}}(t) = \boldsymbol{\eta}\mathcal{E}(t)$, where $\boldsymbol{\eta}$ is a constant *real* unit vector and $\mathcal{E}$ is a scalar amplitude. Find the leading-order approximation for $\tilde{\boldsymbol{B}} = \mathrm{Re}[\boldsymbol{\mathcal{B}}(t)\mathrm{e}^{\mathrm{i}\theta(t,\boldsymbol{x})}]$ from Faraday's law and express $\boldsymbol{\mathcal{E}}' \doteq \boldsymbol{\mathcal{E}} + \boldsymbol{v} \times \boldsymbol{\mathcal{B}}/c$ through $\boldsymbol{\mathcal{E}}$. Show that the corresponding $\mathcal{E}_a'^*\mathcal{E}_b'$ is real for any a and b.

(b) [12 points] Adopt $f = \bar{f}(t, \boldsymbol{v}) + \tilde{f}(t, \boldsymbol{x}, \boldsymbol{v})$, where the bar denotes spatial average, and assume $\tilde{f} \ll \bar{f}$. Derive the linearized equation for $\tilde{f}$ and solve it assuming that $\bar{f}$ is fixed, the wave is monochromatic, and the initial conditions for $\tilde{f}$ are ignorable. (Although inadequate in general, this model will be sufficient.) Show that $\bar{f}$ satisfies

$$\frac{\partial \bar{f}}{\partial t} \approx \frac{\partial}{\partial v_a}\left(D_{ab}\frac{\partial \bar{f}}{\partial v_b}\right) - \nu(\bar{f} - f_0), \qquad D_{ab} = \frac{\nu e^2}{2m^2}\frac{\mathcal{E}_a'^*\mathcal{E}_b'}{(\omega - \boldsymbol{k}\cdot\boldsymbol{v})^2 + \nu^2}, \tag{2}$$

where ω is the wave real frequency and $\boldsymbol{k}$ is the real wavevector. *Hint:* Notice that $\partial_{\boldsymbol{v}} \cdot \boldsymbol{\mathcal{E}}' = 0$ and use the result from **(a)**.

(c) [12 points] Show that in the cold plasma limit $\boldsymbol{k}\cdot\boldsymbol{v} \ll \omega$ and for sufficiently rare collisions $\nu \ll \omega$, Eq. (2) leads to the following equation for the plasma momentum density $\boldsymbol{P}$:

$$\frac{\partial \boldsymbol{P}}{\partial t} = n_0\boldsymbol{F} - \nu\boldsymbol{P}, \qquad \boldsymbol{F} = \frac{\nu e^2}{2m\omega^3}\,\boldsymbol{k}|\mathcal{E}|^2. \tag{3}$$

The vector $\boldsymbol{F}$ can be understood as the wave-driven force per particle. Often (for example, in atomic physics), it is loosely called light pressure or radiation pressure.

(d) [13 points] Assume that the wave is transverse, so in the cold limit it is governed by the dielectric function $\epsilon = 1 - \omega_p^2/[\omega_c(\omega_c + \mathrm{i}\nu)]$. Using the appropriate dispersion relation, find the wave complex frequency $\omega_c = \omega + \mathrm{i}\gamma$ to the first order in ν. Also calculate the wave action density $\mathcal{I}$ and the corresponding wave momentum $\boldsymbol{P}_{\mathrm{w}}$. Using these results and the geometrical-optics equation for $\boldsymbol{P}_{\mathrm{w}}$, show that $\partial_t\boldsymbol{P}_{\mathrm{w}} = -n_0\boldsymbol{F}$.

(e) [4 points] Show that, in the limits used in **(c)**, the transverse-wave momentum *entirely* consists of the electromagnetic-field momentum. Based on the result from **(b)**, qualitatively explain how the wave momentum is absorbed.

I.4: Irreversible Processes Short Problem [10 points]

Show that the Balescu–Lenard operator that describes same-species, singly-charged-particle collisions,

$$C = \frac{\partial}{\partial \boldsymbol{p}} \cdot \int \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3}\, \mathrm{d}\boldsymbol{p}' \, \frac{\pi \boldsymbol{k}\boldsymbol{k}}{|\epsilon(\boldsymbol{k}\cdot\boldsymbol{v}, \boldsymbol{k})|^2} \left(\frac{4\pi e^2}{k^2}\right)^2 \delta(\boldsymbol{k}\cdot\boldsymbol{v} - \boldsymbol{k}\cdot\boldsymbol{v}') \cdot \left(\frac{\partial f(\boldsymbol{p})}{\partial \boldsymbol{p}}\, f(\boldsymbol{p}') - f(\boldsymbol{p})\, \frac{\partial f(\boldsymbol{p}')}{\partial \boldsymbol{p}'}\right),$$

conserves the kinetic-energy density of these species in a homogeneous plasma. Here, $f(\boldsymbol{p})$ is the momentum distribution, $\boldsymbol{v} = \boldsymbol{p}/m$, $\boldsymbol{v}' = \boldsymbol{p}'/m$, and the remaining notation is as usual.

I.5: Applied Math Short Problem [15 points]

(a) **[3 points]** Consider the transcendental equation:

$$x = e^x - \alpha \tag{1}$$

where α and x are real. Show that for $\alpha > 1$ solutions exist? In this case how many solutions exist?

(b) **[5 points]** Consider the iteration

$$x_{n+1} = e^{x_n} - \alpha.$$

for $\alpha > 1$. Give criteria on x_0 for the iteration to converge. Which solution of Eq. (1) does the iteration converge to?

(c) **[7 points]** Construct iteration schemes to find the other real roots of Eq. (1). Outline the constraints on x_0 for convergence of your scheme. Is your convergence linear or quadratic?

I.6: Experimental Methods [45 points]

The current-voltage characteristic (the I-V curve) shown on the next page was obtained from a single-tipped Langmuir probe in an unmagnetized laboratory plasma.

(a) [**5 points**] Identify the ion saturation, electron saturation, and transition regions in the probe curve.

(b) [**5 points**] Which bias conditions define these regions?

(c) [**5 points**] Indicate the approximate values for the floating potential and space potential (on the curve itself, and numerically).

(d) [**5 points**] If the ion species is hydrogen, use the results of (c) to estimate the electron temperature.

(e) [**7 points**] In addition, use the appropriate region of the probe trace to provide an alternative estimate of the electron temperature.

(f) [**8 points**] If the probe tip (the conductor immersed in the plasma) is 1 mm in diameter and 5 mm long, what is the approximate value of the plasma density? (again, assume hydrogen for the ionic species) Assume $T_e \gg T_i$. Are sheath expansion effects likely to affect the accuracy of the density estimate? Explain.

(g) [**5 points**] Discuss any possible changes to the values you obtained in parts (c), (d), (e), and (f) which might result if the plasma is strongly magnetized.

(h) [**5 points**] Other than density, can any information regarding the ion population be obtained from the probe I-V curve?

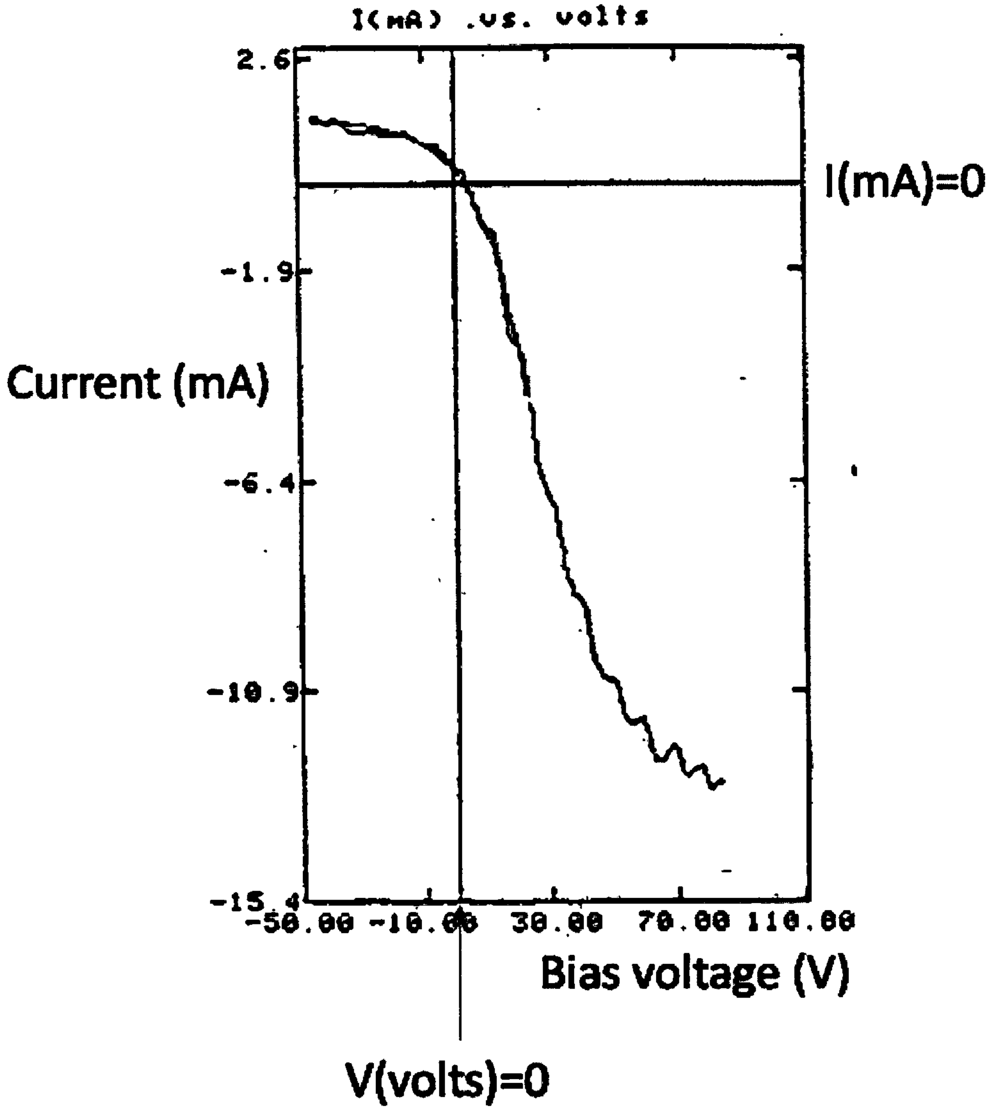

FIG. 1 Langmuir probe characteristics.

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GENERAL EXAMINATION, PART II

May 3, 2022

9:00 a.m. – 1:40 p.m.

- Today's exam (Part II) contains 6 problems on pages 2–7. Answer all problems.
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II.1: General Plasma Physics [25 points]

Consider a point particle whose phase space dynamics is determined by the canonical Hamilton's equation and a given Hamiltonian function $H(\boldsymbol{q}, \boldsymbol{p}, t)$, i.e.,

$$\dot{\boldsymbol{q}} = \frac{\partial H}{\partial \boldsymbol{p}}, \tag{1}$$

$$\dot{\boldsymbol{p}} = -\frac{\partial H}{\partial \boldsymbol{q}}. \tag{2}$$

Here, $(\boldsymbol{q}, \boldsymbol{p})$ is the phase space coordinate. Liouville's theorem states that the dynamics of the particle conserves the phase space volume.

(a) [**5 points**] Equations (1) and (2) describe the dynamics of point (particle) in phase space. What is meant by phase space volume conservation in Liouville's theorem?

(b) [**10 points**] Consider a charged particle in an external electromagnetic field. Let $(\boldsymbol{x}, \boldsymbol{p} \equiv m\dot{\boldsymbol{x}})$ be the phase space coordinate for the particle, and the electromagnetic field is specified by the 4-potential $(\phi(\boldsymbol{x}, t), \boldsymbol{A}(\boldsymbol{x}, t))$. The canonical Hamilton's equation of the charged particle is expressed in terms of $(\boldsymbol{x}, \boldsymbol{P})$, where $\boldsymbol{P} \equiv \boldsymbol{p} + q\boldsymbol{A}/c$ is known as the canonical momentum. The Hamiltonian function is

$$H(\boldsymbol{x}, \boldsymbol{P}, t) = \frac{(\boldsymbol{P} - q\boldsymbol{A}/c)^2}{2m} + q\phi.$$

According to Liouville's theorem, the phase space volume measured in terms of $(\boldsymbol{x}, \boldsymbol{P})$ is conserved. But it turns out that the phase volume measured in terms of $(\boldsymbol{x}, \boldsymbol{p})$ is also conserved. Prove this fact, i.e., the dynamics of the charged particle in the external electromagnetic field conserves the phase space volume measure in terms of $(\boldsymbol{x}, \boldsymbol{p})$.

(c) [**10 points**] For the charged particle considered above, we now conclude that the phase space volumes measured in terms of both $(\boldsymbol{x}, \boldsymbol{p})$ and $(\boldsymbol{x}, \boldsymbol{P})$ are conserved. There has been a debate in accelerator physics community on which volume should be the correct volume when discussing the physics of phase space volume conservation. But the fact is that these two volumes are identical. Prove this fact.

II.2: Waves Short Problem [15 points]

Starting from Maxwell's equations, derive the dispersion relation for the X wave propagating perpendicular to the magnetic field in cold plasma. Identify the equations that determine cutoffs and resonances in terms of R, L, $S = (R+L)/2$, $D = (R-L)/2$, and P that enter the expression for the dielectric tensor ϵ, which you may take for granted:

$$\epsilon = \begin{pmatrix} S & -\mathrm{i}D & 0 \\ \mathrm{i}D & S & 0 \\ 0 & 0 & P \end{pmatrix}. \tag{1}$$

What is the wave polarization near the corresponding resonances?

II.3: MHD [30 points]

Consider an equilibrium sheared magnetic field

$$\boldsymbol{B} = b \tanh\left(\frac{x}{L}\right) \hat{\boldsymbol{y}} + B_0 \hat{\boldsymbol{z}}, \tag{1}$$

where b, L, and B_0 are constants, and other symbols have their usual meanings. Assume that all plasma motion is confined to the $x - y$ plane, and that z remains an ignorable coordinate at all times.

(a) [5 points] Representing the magnetic field in the form $\boldsymbol{B} = \hat{\boldsymbol{z}} \times \nabla\Psi(x, y) + B_0 \hat{\boldsymbol{z}}$, obtain an expression for the flux function $\Psi_0(x, y)$ and current density.

(b) [10 points] Assume that the equilibrium magnetic flux is perturbed by a fluctuation $\Psi_1(x, y, t)$. From the resistive MHD Ohm's law, obtain a time-evolution equation for $\Psi_1(x, y, t)$ in terms of the single-fluid velocity $\boldsymbol{v}$ and the Lundquist number of the plasma. Identify the nonlinear terms in the time-evolution equation.

(c) [10 points] Assume a perturbation of the form $\Psi_1(x, y, t) = \Psi_1(t) \cos ky$, where k is a constant wavenumber. Derive the relation between the island width $W(t)$ and $\Psi_1(t)$, assuming that the islands obey the "constant-Ψ" approximation. What happens to the island width in the ideal MHD limit? Why?

(d) [5 points] Explain *qualitatively* the time-dependence of $W(t)$, linearly and nonlinearly for "constant-Ψ" islands.

II.4: Irreversible Processes [55 points]

(a) [10 points] For a low-density trace species α colliding with a Maxwellian species β, show that the angle-averaged collision operator can be written as

$$C_{\alpha\beta}(f_\alpha) = \frac{1}{v^2}\frac{\partial}{\partial v}\left[v^2 c_1(v)\left(v f_\alpha + c_2 \frac{\partial f_\alpha}{\partial v}\right)\right] \tag{1}$$

where the coefficient c_2 is independent of velocity. (Your starting point can be a result in the NRL formulary.)

(b) [5 points] Solve $C_{\alpha\beta}(f_\alpha) = 0$ for f_α. Give a physical interpretation for the value of c_2. (One or two sentences is enough.)

(c) [10 points] Derive a simple expression for $c_1(v)$ for particles with $v \ll v_{t\beta}$, where $v_{t\beta}$ is the thermal velocity for species β.

(d) [10 points] A simple flux-surface-averaged and pitch-angle averaged quasilinear operator Q describing the effects of second harmonic heating of a species α by ion cyclotron waves is:

$$Q(f_\alpha) = \frac{1}{v^2}\frac{\partial}{\partial v}\left(v^4 \nu_{\mathrm{rf}} \frac{\partial f_\alpha}{\partial v}\right) \tag{2}$$

where ν_{rf} is independent of velocity. Calculate the heating power density P_{rf} in terms of ν_{rf} and the density and average energy of species α. Use this to express ν_{rf} in terms of P_{rf} and other parameters.

(e) [15 points] Find an analytic solution for the steady-state distribution function $f_\alpha(v)$ at high velocity where the RF velocity diffusion is balanced only by drag on electrons (but assume the velocity is still small compared to the electron thermal velocity). (*Hint:* unlike the case of a source from beam injection, in this case the net flux of particles in velocity space in steady state is 0.)

II.5: Experimental Short Problem [15 points]

Consider that a solid material has been heated to a quasi-warm-dense-matter regime where the electron density $n_e \sim 10^{29}\mathrm{m}^{-3}$ and temperature $T \sim 30$ eV. (This temperature is chosen to be slightly above warm-dense conditions, i.e. $T >$ Fermi temperature so that no quantum degeneracy effects need to be considered.) The probe photons have an energy $E_\nu = 5$ keV. A Thomson scattering geometry is defined with a large scattering angle of 150 degrees (near back-scattering, see Figure below). For all the following questions, show a formula and a numerical estimate (i.e. in physical units) for full credit.

(a) **[5 points]** Calculate the numerical magnitude of the Thomson scattering vector $|\boldsymbol{k}|$, given probing photons with incident wavenumber $\boldsymbol{k_i}$ at 5 keV and scattered wavenumber $\boldsymbol{k_s}$, considering $|\boldsymbol{k_i}| \approx |\boldsymbol{k_s}|$.

(b) **[5 points]** Show that this scattering vector will probe the *non-collective* regime.

(c) **[5 points]** Calculate the characteristic Doppler broadening of the scattered light (in units of eV).

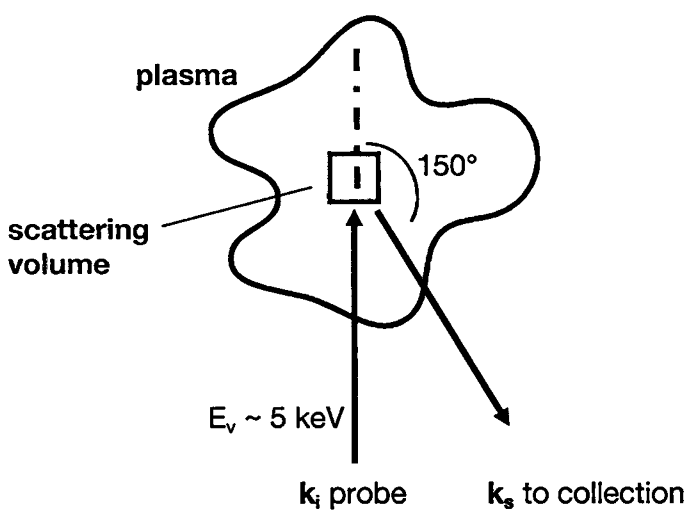

FIG. 1 Experimental setup for Thomson scattering measurement.

II.6: Applied Math [40 points]

The so-called *plasma dispersion function* appears very often in problems involving small-amplitude waves propagating through warm plasmas. Its integral representation is given by

$$Z(\zeta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-t^2}}{t-\zeta} dt \quad (\mathrm{Im}(\zeta) > 0), \tag{1}$$

$$= 2\mathrm{i}\, e^{-\zeta^2} \int_{-\infty}^{\mathrm{i}\zeta} e^{-t^2} dt, \tag{2}$$

$$= \mathrm{i} \int_0^{\infty} e^{\mathrm{i}\zeta t - t^2/4} dt, \tag{3}$$

where Eqs. (2-3) analytically continue Eq. (1) for $\mathrm{Im}(\zeta) \leq 0$ using the *entire error function* with complex arguments. Note also that, by integrating by parts, the plasma dispersion function can be shown to be a solution to the differential equation

$$\frac{dZ}{d\zeta} + 2\zeta Z = -2. \tag{4}$$

This function is known to have the asymptotic expansion for large ζ

$$Z(\zeta) \sim \mathrm{i}\sigma\sqrt{\pi} e^{-\zeta^2} - \frac{1}{\zeta} - \frac{1}{2\zeta^3} - \frac{3}{4\zeta^5} - \frac{15}{8\zeta^7} + \cdots \quad (|\zeta| \to \infty). \tag{5}$$

Here, $\sigma = 0$ for $\mathrm{Im}(\zeta) > 0$, $\sigma = 1$ for $\mathrm{Im}(\zeta) = 0$, and $\sigma = 2$ for $\mathrm{Im}(\zeta) < 0$. Prove this result following the suggested steps, or otherwise. You may need the relations $\Gamma(x+1) = x\Gamma(x)$, $\Gamma(1/2) = \sqrt{\pi}$, $\Gamma(2x) = 2^{2x-1}\Gamma(x)\Gamma(x+1/2)/\sqrt{\pi}$, where $\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-x} dt$ (for $\mathrm{Re}(t) > 0$).

(a) [6 points] Consider the integral in Eq. (3). Is there a saddle point through which you can potentially develop a steepest descent path? Obtain the leading order contribution to the integral from the neighborhood of the saddle point.

(b) [10 points] Determine the steepest descent path needed to evaluate Eq. (3) for large $|\zeta|$ **including** the path through the saddle point. Explain the changes in σ (known as the Stokes phenomenon) using the changes in the path. Does this discontinuity in the asymptotic expressions make sense?

(c) [8 points] Evaluate the integral in Eq. (3) for large $|\zeta|$ using the steepest descent method to prove that the first two terms in the asymptotic expansion in Eq. (5) are correct (i.e. only up to $-1/\zeta$ term).

(d) [10 points] Obtain the full asymptotic expansion of $Z(\zeta)$. You can use Eq. (3), or you can switch to Eqs. (2) or (4) if convenient.

(e) [6 points] The series becomes divergent as typically expected for an asymptotic expansion. Give the optimal truncation number depending on the size $|\zeta|$.