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1 Introduction

An interferometer measures the phase difference between two paths, by interfering the light from one path with the light from the other path. In a plasma, we typically send light through a reference path (with no plasma) and through the plasma. If you remember from the Mach–Zehnder interferometer lab in plasma lab, we start with no plasma, and measure the relative phase difference from the interference (destructive or constructive) as a reference. We then turn on the plasma, and see how much the phase changes. From that phase, we can measure the line-integrated plasma density.

2 Mach-Zender Interferometer

The setup of this problem is shown in figure 1. Each laser beam is split split, sent through their respective path, and recombined at a detector which measures the interference for that particular beam.

At first, the plasma is turned off, and the relative phase between the two paths is measured. This absolute phase doesn't mean anything. The plasma is then turned on, and the relative phase is measured - or rather, the change in phase with the plasma being turned on is measured. However, there are also vibrations of the optical components, which we would like to remove from the measurement. We can write the phase change in each detector as

$$\Delta\Phi_1 = \Delta\Phi_{plasma1} + k_1\delta x_{mech}$$

$$\Delta\Phi_2 = \Delta\Phi_{plasma2} + k_2\delta x_{mech}$$

Our detectors measure $\Delta\Phi_1$ and $\Delta\Phi_2$. We would like to measure $\Delta\Phi_{plasma}$, because we can relate that to the line-integrated density. Subtracting these two equations, we have

$$\frac{\Delta\Phi_2}{k_2} - \frac{\Delta\Phi_1}{k_1} = \frac{\Delta\Phi_{plasma2}}{k_2} - \frac{\Delta\Phi_{plasma1}}{k_1}$$

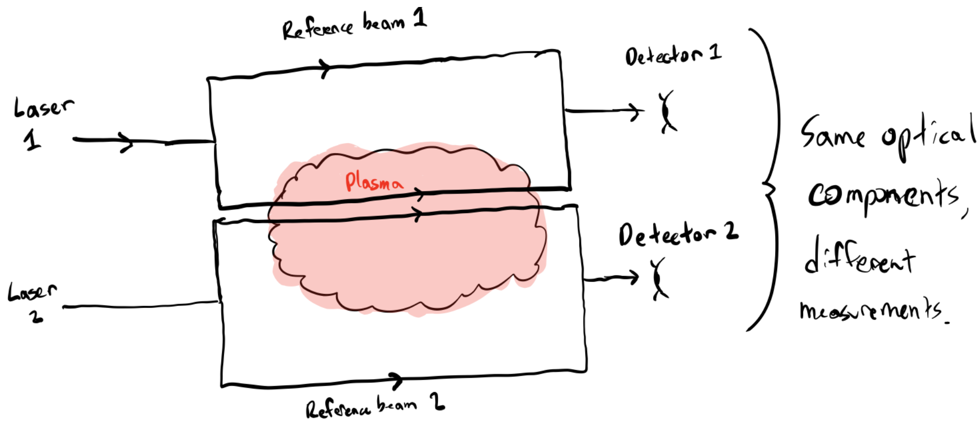

Figure 1: A Mach-Zender interferometer. Reference beam 1 and reference beam 2 share the same optical components, so any mechanical vibrations affect the two beams equally.

We'd like to change our k's to λ's, which we can easily do using $k = \frac{2\pi}{\lambda}$. Cancelling the 2π, we have

$$\lambda_2 \Delta\Phi_2 - \lambda_1 \Delta\Phi_1 = \lambda_2 \Delta\Phi_{plasma2} - \lambda_1 \Delta\Phi_{plasma1} \tag{1}$$

3 Dispersion of Electromagnetic Plasma Waves

Now we need to relate the phase shift to the line-integrated density, for each laser. The total phase accumulated over the path is

$$\Phi = \int k dx$$

The laser frequency (which must be above the plasma frequency) has the dispersion relation $\omega^2 = c^2k^2 + \omega_p^2$, so $k = \sqrt{\frac{\omega^2-\omega_p^2}{c^2}} \approx \frac{\omega}{c}(1 - \frac{\omega_p^2}{2\omega^2}) = \frac{\omega}{c} - \frac{e^2}{2\omega c\epsilon_0 m_e} n_e(x)$. Therefore, the total phase accumulated over the path for each laser is

$$\Phi = \int k dx = \int (\frac{\omega}{c} - \frac{e^2}{2\omega c\epsilon_0 m_e} n_e(x))dx = \text{Const} - \int \frac{e^2}{2\omega c\epsilon_0 m_e} n_e(x)dx$$

Therefore,

$$\Delta\Phi_{plasma} = -\frac{e^2}{2\omega c\epsilon_0 m_e}\int n_e(x)dx$$

Since $c = \lambda f = \lambda\omega/2\pi$, $\omega = 2\pi c/\lambda$, we have

$$\Delta\Phi_{plasma} = -\frac{e^2\lambda}{4\pi\omega c^2\epsilon_0 m_e}\int n_e(x)dx \tag{2}$$

4 Part A

Using equations 1 and 2, and using CO2 and HeNe lasers, we can solve for the line-integrated density.

$$\lambda_C \Delta\Phi_C - \lambda_{He}\Delta\Phi_{He} = -\frac{e^2}{4\pi c^2 \epsilon_0 m_e} \int n_e(x)dx \left[\lambda_C^2 - \lambda_{He}^2\right]$$

$$\int n_e(x)dx = \frac{4\pi c^2 \epsilon_0 m_e}{e^2} \frac{\Delta\Phi_{He}\lambda_{He} - \Delta\Phi_C \lambda_C}{\lambda_C^2 - \lambda_{He}^2} \tag{3}$$

This is our answer.

5 Part B

Suppose that some quantity z is a function of two measured quantities, x and y.

$$z = f(x, y)$$

The uncertainty in z due to the uncertainties in x and y add in quadrature, and are related to the partial derivatives of f with respect to x and y.

$$(\delta z)^2 = (\frac{\partial f}{\partial x})^2(\delta x)^2 + (\frac{\partial f}{\partial y})^2(\delta y)^2$$

However, we are only interested in the uncertainty of the line-integrated density due to our uncertainty about Φ_{He}, which is π.

Since the HeNe laser has a shorter wavelength than the CO2 laser, then it is easier for the HeNe detector to get to a π phase change due to the mechanical vibrations (and plasma dispersion, although my understanding is that in practice the mechanical vibrations lead to larger phase shifts than the plasma). So we can basically ignore the CO2 laser when it comes to our uncertainty, because $\delta(\Delta\Phi_C)$ will be much smaller than $\delta(\Delta\Phi_{He})$.

$$\delta \int n_e(x)dx = \frac{4\pi c^2 \epsilon_0 m_e}{e^2} \frac{\lambda_{He}}{\lambda_C^2 - \lambda_{He}^2} \pi$$

$$\delta n_e(x) = \frac{4\pi c^2 \epsilon_0 m_e}{e^2} \frac{\lambda_{He}}{\lambda_C^2 - \lambda_{He}^2} \frac{\pi}{L} \tag{4}$$

This is our answer.

6 Part C

This just requires dividing equation 4 by the density n_e, and plugging in numbers.