2018 II.2

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If

$$\frac{d}{dt}(\frac{1}{\omega^2}) = A$$

where A is a constant, then

$$\frac{1}{\omega^2} = At + B \tag{1}$$

where B is also a constant.

We would like to see if the observed FRB behavior is consistent with the hypothesis that these signals are due to wave dispersion in the interstellar medium. Since the interstellar medium is low-density, then at radio frequencies $\omega^2 \gg \omega_{pe}^2$. Since the interstellar medium is low-B, then $\omega^2 \gg \Omega_e^2$ at radio frequencies. Therefore any waves traveling through interstellar space will be electromagnetic plasma waves, which as we know have the dispersion relation

$$\omega^2 = c^2k^2 + \omega_p^2$$

Now, we would like to calculate the group velocity of these waves as a function of ω, because if we have the group velocity of these waves then we can calculate the transit time of these waves across the universe as a function of ω. This is because, using the definition $v_g = \frac{dx}{dt}$,

$$\int dt = t = \int \frac{1}{v_g} dx$$

Using the dispersion relation, we can figure out v_g.

$$\frac{d}{dk}(\omega^2) = 2\omega\frac{d\omega}{dk} = 2c^2k$$

$$v_g = \frac{d\omega}{dk} = \frac{c^2k}{\omega} = \frac{c\sqrt{\omega^2 - \omega_p^2}}{\omega}$$

With v_g in hand, we can now solve for the transit time $t(\omega)$.

$$t(\omega) = \int \frac{1}{v_g} dx = \frac{1}{c}\int \frac{1}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}}} dx \approx \frac{1}{c}\int (1 + \frac{\omega_p^2}{\omega^2}) dx = \frac{L}{c} + \frac{1}{c\omega^2}\int \omega_p^2 dx \tag{2}$$

Solving for $\frac{1}{\omega^2}$, we see that equation 2 is in the form of equation 1. Therefore, the FRB behavior is explained by dispersion of electromagnetic plasma waves in the interstellar medium.