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Generals Notecards, May 11, 2016

GPP I

- Space- and time-scale definitions
- Plasma parameter
- Quick derivations of Debye shielding and Langmuir oscillations
- Single particle motion: drifts, adiabatic invariants, and mirror confinement
- ~~Lawson criterion and fusion cross-section~~
- Child-Langmuir Law

MHD

- Magnetic fields as a Hamiltonian system
- MHD equations (8)
- Regimes of MHD validity
- Equilibria: force-free, Grad-Shafranov
- Island width
- Stability: energy principle, Rayleigh-Taylor
- MHD waves: dispersion relation and properties of waves
- Alfven's theorem of frozen-in flux
- Linear and non-linear tearing instability
- Sweet-Parker model of reconnection
- MHD turbulence, correlations and scaling laws
- Mean-field dynamo theory

Neoclassical

- Heuristics: parallel velocity and fraction of trapped particles
- Banana orbit width (two methods)
- Diffusion: banana, plateau, Pfirsch-Schluter
- Ware pinch: flux of trapped particles
- Bootstrap current

Diagnostics

- Langmuir probes, double and triple probes
- Thomson scattering cross-section
- Interferometry
- Magnetic diagnostics and confinement time
- (review all homeworks)

Asymptotics

- Basic methods: Wronskian, variation of parameters, Green's functions
- Fuch's classification and Frobenius expansions for regular singular points
- Asymptotic balance to find leading behavior

- Integral representations: Fourier-Laplace kernel
- Phase integral methods and rules, flux in WKB
- Boundary layer problems, multiscale analysis

Waves

- Real frequency waves: field equation, HSM, group/phase velocity, dispersion relation
- Complex frequency waves: Laplace transform, initial conditions, Landau's solution of quasimodes
- Manual for finding a dispersion relation
- Kramers-Kronig relations: dispersive media are always dissipative (conductivity must be complex)
- Solution to a driven Schrodinger equation
- Dielectric tensor and dispersion relation for cold unmagnetized waves
- Stix's calculation of cold magnetized dielectric tensor
- Low-frequency limit for shear and compressional Alfven waves
- Cold magnetized dispersion relation: resonances and cutoffs, plot of N vs. frequency
- Necessary and sufficient conditions for electrostatic waves
- Parallel propagating waves: R and L waves and their properties/polarizations
- Perpendicular propagating waves: O and X waves
- Tokamak parameters: overdense/underdense (ω/Ω_e), c/V_A
- Hydrodynamic approach: momentum equation, pressure model, continuity equation
- Dielectric tensor, $\epsilon_\perp, \epsilon_\parallel$, longitudinal waves at high-frequency (dispersive Langmuir waves) and low-frequency (Debye shielding, IAW)
- Kinetic approach: Liouville and Vlasov equations
- Conductivity for electrostatic waves, phase mixing
- Plasma dispersion function for a Maxwellian
- Landau rule and kinetic equation for susceptibility, and differences from cold-plasma case
- Nyquist theorem, number of unstable modes, proving single-peaked distributions are stable
- Weakly dissipative waves, equation for ω_i, derivation of damping for Langmuir waves
- Single-particle-motion picture of Landau damping, wave amplification and damping, regimes of applicability
- BGK waves' frequency shift
- Dielectric tensor for electromagnetic kinetic waves in non-magnetized plasma from Vlasov equation, effects on longitudinal and transverse waves
- Warm magnetized plasmas: integrating along characteristics due to non-zero $\overrightarrow{F_0}$ in linearized Vlasov equation
- Conductivity $\Delta j/\Delta E$ and integration over time and space to get magnetized χ_s
- Dimensionless parameters $\lambda = \frac{k_\perp^2 w^2}{2\,\Omega^2}$, $\xi_n = \frac{\omega - n\Omega}{k_\parallel w}$ and fluid limit
- Limits of Bessel functions in limits of parallel and perpendicular propagation, X/O-wave limits
- Electron Bernstein waves: electrostatic limit $\epsilon_{xx} = 0$, real-frequency dispersion relation
- Mode conversion between EBW, DK and X-wave: tokamak heating with X-waves converted to EBWs near the upper-hybrid resonance
- Geometrical optics for non-HSM waves, using small parameter $\epsilon = \max\left\{\frac{\tau}{T}, \frac{l}{L}, \frac{d\tau}{dt}, \frac{dl}{dx}\right\}$
- Action principle and derivation of ponderomotive potential and wave Lagrangian
- Generalization to magnetized case, Hermitian and anti-Hermitian parts of α
- $K - \chi$ theorem and the relation between α and susceptibility
- Dielectric tensor in wave Lagrangian density

- Whitham's equations (variations of Lagrangian), consistency relations for ω and k, and the action conservation theorem, and formula for the action density
- Ray equations and quantum analogy with Hamilton's equations
- Transport equations for energy, momentum as corollaries of the action conservation theorem
- Poynting vector, kinetic energy flux, and a formula for the group velocity
- Understanding dissipation in terms of the transport equations and the anti-Hermitian part of the dielectric tensor
- Applications to power dissipation in Landau damping, $P_{zz,n=0}$, and relationships to ω_i
- Transit-time magnetic pumping with E_{eff} and $P_{yy,n=0}$
- Cyclotron damping, $|E_+|^2$ and $|E_-|^2$, plus the non-linear picture of saturation (energy conservation in wave frame)
- Quasilinear theory: Chirikov criterion, diffusion coefficient in terms of spectral density of the electric-field energy, U_k
- Time-evolution equation for U_k and *linear* dispersion relation for ω_k→ quasilinear theory
- Conservation of particles, momentum, and energy from quasilinear equations
- Application of quasilinear theory to bump-on-tail instability

Irreversibles

- Bogoliubov hierarchy of timescales
- Ensemble averaging, 2-point correlation functions, diffusion coefficients and Taylor's formula
- Langevin equation and standard analysis
- Fourier power spectra in statistical homogeneity
- Fokker-Planck equation from Taylor-expanding the Markovian master equation
- Collisions and the full form of Fokker-Planck: Landau form and Rosenbluth potentials
- Conservation properties of the Landau form
- Isotropic limit of the collision operator and energetic particle slowdown
- Liouville, Klimontovich and Vlasov equations
- The information approach to entropy and maximization of entropy with Maxwellians
- BBGKY hierarchy and the two-particle correlation function $g_{1,2}(\rho)$
- Fluctuation spectrum in thermal equilibrium
- Test-particle dynamics and drag; superposition principle to get electric field spectrum
- Chapman-Enskog methodology and derivation of Braginskii's fluid equations