

Wavelength II.2 Waves

1/2

Disp. relation for X wave

$$\nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t}, \quad \nabla \times B = \frac{1}{c} (\frac{\partial}{\partial t} \epsilon E) \quad (j_s = 0)$$

$$(\nabla \times (\nabla \times E)) = -\frac{\partial}{\partial t} \left(\frac{1}{c^2} \frac{\partial}{\partial t} (\epsilon E) \right)$$

$$\frac{c^2}{\omega^2} (k \cdot E) k - \frac{k^2 c^2}{\omega^2} E + \epsilon E = 0$$

$$\epsilon = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix}$$

$$\left(\frac{c^2}{\omega^2} |k\rangle \langle k| - \frac{c^2}{\omega^2} \langle k|k\rangle \mathbb{I} + \epsilon \right) \vec{E} = 0 \quad \text{define } N = \frac{kc}{\omega}$$

$$(|N\rangle \langle N| - N^2 \mathbb{I} + \epsilon) \vec{E} = 0 \quad \begin{matrix} \text{general dispersion} \\ \perp \text{propagation to } \hat{B} = \hat{z} \end{matrix}$$

D_E

$$= \begin{pmatrix} S & -iD & 0 \\ iD & S-N^2 & 0 \\ 0 & 0 & P-N^2 \end{pmatrix} E \quad \begin{matrix} P-N^2 \text{ is } 0 \text{ wave disp.} \\ \text{relation} \end{matrix}$$

$$\det \begin{pmatrix} S & -iD \\ iD & S-N^2 \end{pmatrix} = 0 \quad \text{is } X \text{ wave disp. relation}$$

$$S(S-N^2) - D^2 = 0$$

$$S^2 - SN^2 - D^2 = 0 \rightarrow \frac{1}{4} (R^2 + 2RL + L^2) - SN^2 - \frac{1}{4} (D^2 - 2RL + L^2) = 0$$

$$\rightarrow RL - SN^2 = 0 \quad \text{or} \quad N^2 = \frac{RL}{S} \quad \begin{matrix} \text{dispersion} \\ \text{relation} \end{matrix}$$

Now to determine cutoffs / resonance



Wavelength II, J Waves

Resonance when $N^2 \rightarrow \infty$

Cutoff when $N^2 \rightarrow 0$

So, a cutoff can be identified as solutions to $RL = 0$
should result in two solutions

Resonances can be identified with $S=0$, which has two solutions for an electron/ion plasma called the upper hybrid ($\omega_{uh} = \sqrt{\omega_{pe}^2 + \Omega_e^2}$) and lower hybrid with resonances.

Near the resonance, X waves are approximately electrostatic and therefore linearly polarized. They are known as electron/ion Bernstein waves.

