2008 Part 2 Question 5 (MHD)

a. $H = \int A \cdot B \, d^3x$

$\dot{H} = \int \dot{A} \cdot B + \int A \cdot \dot{B}$

$= -\int (\nabla\phi + E) \cdot B - \int A \cdot \nabla \times E$

$= -\int \nabla\phi \cdot B - \int E \cdot B + \int \nabla \cdot (A \times E) - \int E \cdot \nabla \times A$

~~$-2\int E \cdot B$~~

$\int -\nabla \cdot (\phi B) + \phi \nabla \cdot B$ (→ 0) $- 2\int E \cdot B$ (→ 0) $+ \int \nabla \cdot (A \times E)$

(ideal plasma $E + v \times B = 0$)

$-\oint_S \phi B \cdot d\vec{a} + \oint (A \times E) \cdot da = 0.$

0 on conducting surface ($B \cdot n = 0$)

$\oint A \cdot (E \times d\vec{a})$ → 0 on surface ($E \times n = 0$)

b. 1. $\frac{Q_\perp^2}{\mu_0} \geq 0$

2. $\frac{B^2}{\mu_0} (\nabla \cdot \xi_\perp + 2\xi_\perp \cdot \kappa)^2 \geq 0$

3. $\gamma P (\nabla \cdot \xi)^2 \geq 0$

4. $-2(\xi_\perp \cdot \nabla P)(\xi_\perp^* \cdot \kappa)$ ~~any sign~~ 0

5. $-\mu(\vec{r})(\xi_\perp^* \times B) \cdot Q_\perp$