2009 Day 2 Q1 (GPP)

a. $\int (\nabla \times E)\cdot da = \frac{-1}{c}\int \dot{B}\cdot da = \frac{-\pi r^2}{c}\dot{B}$

$\oint E\cdot dl = 2\pi r E_\phi = \frac{-\pi r^2}{c}\dot{B} \rightarrow E = -\frac{1}{2c}\dot{B}\, r\,\hat{\phi}$

Note $\hat{\phi} = \hat{y}\cos\phi - \hat{x}\sin\phi = \hat{y}\left(\frac{x}{r}\right) - \hat{x}\left(\frac{y}{r}\right)$

$\vec{E} = \frac{-\dot{B}}{2c}\left(x\hat{y} - y\hat{x}\right)$

b. $\begin{pmatrix}\ddot{x}\\ \ddot{y}\end{pmatrix} = \frac{q}{m}\begin{pmatrix}E_x\\ E_y\end{pmatrix} + \frac{qB}{mc}\begin{pmatrix}v_y\\ -v_x\end{pmatrix}$

$= \frac{\dot{B}q}{2mc}\begin{pmatrix}y\\ -x\end{pmatrix} + \frac{qB}{mc}\begin{pmatrix}-\dot{y}\\ \dot{x}\end{pmatrix}$

$= \ddot{\theta}\begin{pmatrix}y\\ -x\end{pmatrix} + 2\dot{\theta}\begin{pmatrix}-\dot{y}\\ \dot{x}\end{pmatrix}$

$\rightarrow \ddot{x} - 2\dot{\theta}\dot{y} - \ddot{\theta}y = 0$

$\ddot{y} + 2\dot{\theta}\dot{x} + \ddot{\theta}x = 0$

c. Let $R = X + iY$, $r = x + iy$, $R = re^{-i\theta}$

$R' = r'e^{-i\theta} - i\theta' re^{-i\theta}$

$R'' = r''e^{-i\theta} - 2i\theta' r' e^{-i\theta} - i\theta'' re^{-i\theta} - \theta'^2 re^{-i\theta}$

$R'' + R\Omega^2 = r''e^{-i\theta} - 2i\theta' r' e^{-i\theta} - i\theta'' re^{-i\theta}$

complex conjugate: $(R'' + R\Omega^2)^* = r''^* e^{i\theta} + 2i\theta' r'^* e^{i\theta} + i\theta'' r^* e^{i\theta}$

add eqns: $X'' + \Omega^2 X = x''\cos\theta + 2i\theta' iy \cos\theta + i\theta'' iy\cos\theta = 0$

subtract eqns: $Y'' + \Omega^2 Y = y''\sin\theta + 2\theta'\sin\theta X + 2\theta'' X\sin\theta = 0$

d. $(XY' - YX')' = XY'' - X'Y' - YX'' - Y'X'$

$= X(-\Omega^2 Y) - Y(-\Omega^2 X) = 0.$

Hence $XY' - YX' =$ constant is conserved.