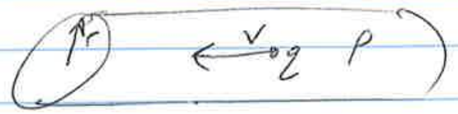
2009, Part 1, Q6 (GPP1)



$$\nabla \cdot E = 4\pi\rho$$

$$\oint E \cdot da = 4\pi \int \rho \, dV$$

$$2\pi r \ell E = 4\pi (\pi r^2) \ell \rho$$

$$E_r = 2\pi\rho r$$

$$\nabla \times B = \frac{4\pi}{c} J$$

$$\oint B \cdot d\ell = \frac{4\pi}{c} \int J \cdot da$$

$$2\pi r B = \frac{4\pi}{c} (\pi r^2) J$$

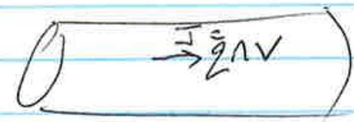
$$B_\phi = \frac{2\pi J r}{c} = \frac{2\pi \rho v r}{c}$$

$$F = q\left(\frac{v \times B}{c} + E\right)$$

$$\frac{F_M}{F_E} = \frac{(v \times B)/c}{E} = \frac{v^2}{c^2} = \beta^2$$

(Radial) electric force and (anti-radial) magnetic force may only be comparable as $\beta \to 1$ ξ relativistic limit.

2009 Day 1 Q6 (GPP)



$\int \nabla\cdot E = 4\pi \int \rho$ $\qquad$ $\int \nabla\times B\cdot da = \frac{4\pi}{c}\int J\cdot da$

$\oint E\cdot da = 4\pi \int \rho$ $\qquad$ $\oint B\cdot dl = \frac{4\pi}{c}\int J\cdot da$

$2\pi r \ell E = 4\pi (nq)\pi r^2 \ell$ $\qquad$ $2\pi r B_\phi = \frac{4\pi}{c} nqv\,\pi r^2$

$E = 2\pi (nq) r\,\hat r \quad r < R_0$ $\qquad$ $B_\phi = \frac{2\pi}{c}(nq) v r \quad r < R_0$

$2\pi r \ell E = 4\pi (nq)\pi R^2 \ell$

$E = \frac{2\pi (nq) R^2}{r}\hat r \quad r > R_0$ $\qquad$ $B_\phi = \frac{2\pi}{c}\frac{(nq) v R^2}{r} \quad r > R_0$

$\cancel{E\times B =}\ \hat r\times\hat\phi$ $\qquad$ $F = q\left(\frac{v\times B}{c} + E\right)$ $\qquad$ $v\times B = \hat z\times\hat\phi = -\hat r, \; E = \hat r$

$\frac{F_E}{F_B} \sim \frac{cE}{vB} = \frac{c}{v}\,\frac{2\pi(nq)r}{(2\pi/c)(nq)v r} \sim \left(\frac{c}{v}\right)^2 \gg 1$

$F_B\,\hat r \sim 2\pi(nqr)\left(\frac{v}{c}\right)^2 = F_E\left(\frac{v}{c}\right)^2$

nonrelativistic → radial (E) force much stronger than antiradial (B) force.