2010 Day 2 Q4 (MHD)

1. When B fields much larger than E fields? $E \propto \frac{V_A}{c} B$
Due to very strong Debye shielding?

$Q = \xi \times B$

2. $\delta W = \frac{1}{2\mu_0}\int 2B \cdot \delta B = \frac{1}{\mu_0}\int B \cdot \nabla \times Q = \frac{1}{\mu_0}\int \nabla\cdot(B\times Q) + Q\cdot\nabla\times B$
$=$ ~~$\frac{1}{\mu_0}\oint Q$~~ $= \frac{1}{\mu_0}\int Q\cdot\nabla\times B = \frac{1}{\mu_0}\int(\xi\times B)\cdot J = \int(B\times J)\cdot\xi$
For δW to vanish for any arbitrary ξ, must have $J \times B = 0$.
Note above we also had $\int\nabla\cdot(B\times Q) = 0$ since ~~$B\times Q = B\times(\xi\times B)$~~
~~$= \oint B\times(B\times\xi$~~ $= \oint(B\times(\xi\times B))\cdot da$ and $\xi\to 0$ on surface.

3. $\delta(W - \lambda K) = 0 = \int \frac{1}{\mu_0} B\cdot\delta B - \lambda A\cdot\delta B - \lambda B\cdot\delta A$
$= \int\frac{1}{\mu_0} B\cdot(\nabla\times\delta A) - \lambda A\cdot(\nabla\times\delta A) - \lambda B\cdot\delta A$
$= \frac{1}{\mu_0}\oint(\delta A\times B)\cdot da - \lambda\oint(\delta A\times A)\cdot da$
$+ \int\frac{1}{\mu_0}(\nabla\times B)\cdot\delta A - \lambda(\nabla\times A)\cdot\delta A - \lambda B\cdot\delta A$
Surface integrals vanish since $\delta A\to 0$ on boundary.
$= \int(J - 2\lambda B)\cdot\delta A$
$\rightarrow\ J = 2\lambda B$, also force-free since $\nabla P = J\times B = 2\lambda B\times B = 0$.

? 4. Both have $J\times B = 0$, so both are force-free.
Differences: λ constant in (3) vs $\lambda(\vec{r})$ in (2)

? 5 Less stable? Harder to achieve?
Need ∇P to confine plasma. ($P\to 0$ at wall, $P\neq 0$ at center)