

2010 Day 1 Q1B (Asymptotics)

$$1 + \epsilon x - \epsilon x^2 + \epsilon^2 x^3 = 0$$

(0) $1 + \epsilon x \approx 0$
 $x_{n+1} =$

(1) $1 - \epsilon x^2 \approx 0$

$$1 - \epsilon x_{n+1}^2 = -\epsilon x_n - \epsilon^2 x_n^3$$

$$\frac{1}{\epsilon} + x_n(1 + \epsilon x_n^2) = x_{n+1}^2$$

$$x_{n+1} = \pm \sqrt{\frac{1}{\epsilon} + x_n(1 + \epsilon x_n^2)}$$

(2) $-\epsilon x^2 + \epsilon^2 x^3 \approx 0$ $x=0$ clearly not solution, eliminated.

$$x \approx \frac{1}{\epsilon}$$

$$1 + \epsilon x = \epsilon x^2 - \epsilon^2 x^3$$

$$\frac{1}{\epsilon} + \frac{1}{x} = 1 - \epsilon x$$

$$\rightarrow x_{n+1} = \frac{1}{\epsilon} - \frac{1}{\epsilon^2 x_n^2} - \frac{1}{\epsilon x_n}$$

