2011 Day 2 Q1B (MHD)

a. In each species,

$$\frac{\partial n}{\partial t} + \nabla\cdot(n v) = 0 \qquad \text{continuity}$$

$$m\frac{dv}{dt} = q\left(E + \frac{v\times B}{c}\right) - \frac{\nabla P}{n} - \frac{R}{n}$$

$$\frac{d}{dt}\left(\frac{P}{n^{\gamma}}\right) = 0$$

b. sum 1-species equations:

$$m_e n_e \frac{dv_e}{dt} + m_i n_i \frac{dv_i}{dt} = \cancel{R_e} - en_e\left(E + \frac{v_e\times B}{c}\right) + en_i\left(E + \frac{v_i\times B}{c}\right) - \nabla P^{tot} - R^{tot}$$

Define $\rho = m_e n_e + m_i n_i \approx m_i n_i = m_i n$

With quasineutrality and singly-charged ions, $n_e = n_i \equiv n$

$$U = \frac{\rho_e v_e + \rho_i v_i}{\rho_e + \rho_i} \approx v_i$$

$$\rightarrow \rho\frac{du}{dt} = \frac{J\times B}{c} - \nabla P - \cancel{\Sigma R}^{0} \quad ✓$$

where $J = \cancel{-en_e v_e}\ e(n_i v_i - n_e v_e)$, $P = P_e + P_i$

To get Ohm's Law, $R_e = en_e\eta J$

and write $en_e v_e = -J + en_i v_i = -J + enU$

$$\rightarrow \rho_e\frac{dv_e}{dt} = -en_e\left(E + \frac{v_e\times B}{c}\right) - \nabla P + R_e$$

$$= -en_e E - (-J + enU)\times\frac{B}{c} - \nabla P + en_e\eta J$$

neglecting small mass.

$$E + \frac{U\times B}{c} - \eta J = \frac{J\times B}{cen} - \frac{\nabla P}{en} - \cancel{\frac{m_e}{e}\frac{dv_e}{dt}}^{0}$$

? c. This may be reduced to general Ohm's Law when RHS $\ll$ LHS. Namely ...

$$\frac{J\times B}{cen}\frac{c}{U\times B} \approx \frac{J}{enU} \cong \frac{en(v_e - v_i)}{en\, v_i} \ll 1,$$ so must have $v_e \approx v_i$, or $T_e \ll T_i$.