2011 Day 2 Q2 (Waves)

(O/X transverse waves occur for $k \perp B_0$)

a. Cold, magnetized plasma, with $B = B_0 \hat{z}$, $k = k\hat{x}$

$$\epsilon = \begin{pmatrix} S & -iD & \\ iD & S & \\ & & P \end{pmatrix}$$

$S = \frac{1}{2}(R+L)$, $D = \frac{1}{2}(R-L)$

$R = 1 - \sum_s \frac{\omega_{ps}^2}{\omega(\omega+\Omega_s)}$, $L = 1 - \sum_s \frac{\omega_{ps}^2}{\omega(\omega-\Omega_s)}$

$P = 1 - \frac{\omega_p^2}{\omega^2}$

$$D = N_i N_j - N^2 \delta_{ij} + \epsilon_{ij}$$

$$= \begin{pmatrix} S & -iD & \\ +iD & S - N^2 & \\ & & P - N^2 \end{pmatrix}$$

$N^2 = P$ is the O-mode (same as unmagnetized transverse EM wave)

X mode obeys $S(S-N^2) - D^2 = 0$

$$N^2 = \frac{S^2 - D^2}{S} = \frac{RL}{S}$$

Resonance at $N^2 \to \infty$, so $S \to 0$ (can check RL remain finite)

$S = \frac{1}{2}\left(2 - \sum_s \frac{\omega_{ps}^2}{\omega^2 - \Omega_s^2}\right)$

$$S = 1 - \frac{\omega_{pe}^2}{\omega^2 - \Omega_e^2} - \frac{\omega_{pi}^2}{\omega^2 - \Omega_i^2}$$

At large enough ω, ion response negligible $\to$ $\omega^2 = \omega_{pe}^2 + \Omega_e^2$

b.

$$\begin{pmatrix} S & -iD \\ iD & S - N^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = 0$$

$$\frac{iE_x}{E_y} = \frac{N^2 - S}{D} \gg 1$$, so polarization approximately aligned with k.

(near UH Resonance)