2011 Day 2 Q4 (MHD)

a. $B = B_0 \nabla\phi = \frac{B_0}{r}\hat{\phi}$

There will be a vertical ∇B drift: $\nabla B = -\frac{B_0}{r^2}\hat{r}$,

$V_{\nabla B} \propto B \times \nabla B = -\hat{\phi} \times \hat{r} = \hat{z}$

b. $J \times B = J_\perp \times B = \nabla P$

$$\begin{pmatrix} -J_z B_\phi \\ J_r B_\phi \end{pmatrix} \begin{matrix} \hat{r} \\ \hat{z} \end{matrix} = \nabla P_\perp$$

$$\nabla \times (J_\perp \times B) = J_\perp (\cancel{\nabla \cdot B}) - B(\nabla \cdot J_\perp) + (B \cdot \nabla) J_\perp - (J_\perp \cdot \nabla) B = \nabla \times \nabla P = 0$$

$$\rightarrow B^2 (\nabla \cdot J_\perp) = B \cdot (B \cdot \nabla) J_\perp - B \cdot (J_\perp \cdot \nabla) B$$

$$((B \cdot \nabla) J_\perp)_\phi = \frac{B_\phi}{r} J_r \qquad ((J_\perp \cdot \nabla) B)_\phi = J_r \frac{\partial B_\phi}{\partial r}$$

$$\rightarrow B^2 (\nabla \cdot J_\perp) = B_\phi \left(\frac{B_\phi}{r} J_r\right) - B_\phi \left(J_r \frac{\partial B_\phi}{\partial r}\right) = \frac{2 B_\phi^2}{r^3} J_r$$

$$\nabla \cdot J_\perp = \frac{2 J_r}{r}$$

From above, we have $J_r B_\phi = \partial P / \partial z$, leaving

$\nabla \cdot J_\perp = \frac{2}{B_0} \frac{\partial P}{\partial z}$. This shows there is a net flow out of the device if there is only a purely toroidal axisymmetric field.

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c. Adding a toroidal current $J_\parallel$ will generate a poloidal field to add to the toroidal one. If suitably chosen, $B \times \nabla B$ can change into a less deleterious direction (e.g. toroidally or azimuthally instead of vertically or radially).

Effect on MHD Equations:

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7. d. $J_{\parallel} = \lambda(\vec{r})\, B$

$B = \frac{B_0}{r}\hat{\phi} + B_p$

Demanding $\nabla \cdot J = 0$, $\nabla \cdot (J_{\parallel} + J_{\perp}) = 0$