2011 Day 1 Question 3 (Diagnostics)

a. Since this is a gradient, perhaps we can assume the differential cross section

$$\frac{d\sigma}{d\Omega} = r_e^2 \sin^2\theta$$

$$\sigma_{tot} = \int \frac{d\sigma}{d\Omega}\, d\Omega = 2\pi r_e^2 \int_0^\pi \sin^3\theta\, d\theta$$

$$\sin^3\theta = \left(\frac{1}{2i}\right)^3 \left(e^{i\theta} - e^{-i\theta}\right)^3 = \left(\frac{1}{2i}\right)^3 \left(e^{3i\theta} - 3e^{i\theta} + 3e^{-i\theta} - e^{-3i\theta}\right)$$

$$= -\frac{1}{4}\left(\frac{1}{2i}\right)\left(2i\sin(3\theta) - 3\cdot 2i\sin\theta\right) = \frac{1}{4}\left(3\sin\theta - \sin 3\theta\right)$$

$$\sigma_{tot} = \frac{\pi}{2} r_e^2 \int_0^\pi 3\sin\theta - \sin 3\theta = r_e^2 \frac{\pi}{2}\left[-3\cos\theta\Big|_0^\pi + \frac{1}{3}\cos 3\theta\Big|_0^\pi\right]$$

$$= r_e^2 \frac{\pi}{2}\left[6 - 2/3\right] = \frac{8\pi}{3} r_e^2 = \frac{8\pi}{3} \times 10 \quad \frac{2.8^2 8\pi}{3} \times 10^{-30}\ m^2$$

$$\sigma_{tot} \approx 8^2\ E{-}30\ m^2 \approx 6.4\ E{-}29\ m^2$$

b. $\Lambda_{scatt} = \Lambda\, \Delta x$

$$N_{scatt} = N_{inc}\, \Lambda_{scatt}\, \sigma_{tot} \left(\frac{.01}{4\pi}\right)$$

$$\frac{N_s}{N_{inc}} = \left(2\ E20\ m^{-3}\right)\left(E{-}2\ m\right)\left(6.4\ E{-}29\ m^2\right)\frac{(E-2)}{12}$$

$$\approx 10^{-13}$$

As starting (a) somewhat earlier, ~~Larmor formula $\frac{dP}{d\Omega} = \frac{q^2 a^2}{4\pi c^3}$~~

Electric dipole radiation has fields $E_{rad} \approx \frac{1}{c^2}\frac{\sin\theta}{r}\ddot{p} \sim B_{rad}$

where p = dipole moment. $\ddot{p} = q\ddot{d} = q\left(\frac{q}{m} E_{in}(t)\right)$

$$\langle S_{rad}\rangle = \frac{4\pi}{c} E_{rad}^2 = \frac{4\pi}{c}\left(\frac{q^2}{mc^2}\right)^2 \frac{\sin^2\theta}{r^2}\left\langle E_{in}^2(t)\right\rangle$$

$$P = \oint \langle S\rangle \cdot da \rightarrow \frac{dP}{d\Omega} = \langle S\rangle \cdot \hat{r}\, r^2 = \frac{4\pi}{c}\left\langle E_{in}^2(t)\right\rangle r_e^2 \sin^2\theta$$

$$\frac{d\sigma}{d\Omega} = \frac{dP/d\Omega}{\langle S_{in}\rangle} = \frac{dP/d\Omega}{\frac{4\pi}{c}\left\langle E_{in}^2(t)\right\rangle} = r_e^2 \sin^2\theta.$$

where $r_e = \frac{q^2}{mc^2}$