Remaining: diagnostics guide (τ_E measurement)
irreversibles long (Taylor formula, hard)
MHD long (Kelvin-Helmholtz instability)

2012 Day 2, Question 1B (Waves)

a. These are thermal corrections to plasma oscillations:

$$\omega^2 = \omega_p^2 + \gamma V_t^2 k^2.$$ low frequency: isothermal, $\gamma = 1$.

For ω very small,

$$k^2 = -\frac{\omega_p^2}{V_t^2} = -\frac{1}{\lambda_D^2} \rightarrow k = \frac{\pm i}{\lambda_D}.$$

b. $E(x,t) \sim E_0 e^{-i(\omega t - kx)} \sim E_0 e^{-k|x|/\lambda_D}$

this ~~repres~~ describes Debye shielding.

If you forgot (a), you could derive from fluid equations. In each species:

sum over species

$$\begin{cases} \frac{\partial n}{\partial t} + \nabla\cdot(n v) = 0 & \rightarrow \frac{\partial \tilde{n}}{\partial t} + n_0 \nabla\cdot\tilde{v} = 0 \\ m\frac{dv}{dt} = qE - \frac{\nabla P}{n} & \rightarrow m\frac{\partial \tilde{v}}{\partial t} = q\tilde{E} - \frac{\nabla\tilde{P}}{n_0} = -q\nabla\tilde{\phi} - \frac{\nabla\tilde{P}}{n_0} \\ \nabla^2\phi = -4\pi q n & \nabla^2\tilde{\phi} = -4\pi q\tilde{n} \\ \frac{d}{dt}\left(\frac{P}{n^\gamma}\right) = 0 & \tilde{P} = \gamma T_0 \tilde{n} \quad (\gamma = 1,\ \text{isothermal}) \end{cases}$$

$$\Rightarrow \frac{\partial}{\partial t}\nabla\cdot\tilde{v} = -\frac{q}{m}\nabla^2\phi - \frac{\nabla^2\tilde{P}}{mn_0}$$

$$-\frac{\partial^2}{\partial t}\left(\frac{\tilde{n}}{n_0}\right) = \frac{4\pi q^2}{m}\tilde{n} - \nabla^2\left(\frac{T_0}{mn_0}\right)\tilde{n}$$

$$\left(-\partial_t^2 + V_t^2\nabla^2 - \omega_p^2\right)\tilde{n} = 0$$

Fourier-Transform: $+\omega^2 - k^2V_t^2 - \omega_p^2 = 0 \rightarrow \omega_{pn}^2 = \sum_s \omega_{ps}^2 + k^2 V_{ts}^2$

$$= \omega_p^2 + k^2V_t^2$$

$$= 1 + k^2\lambda_D^2.$$

For hot limit, use plasma dispersion instead