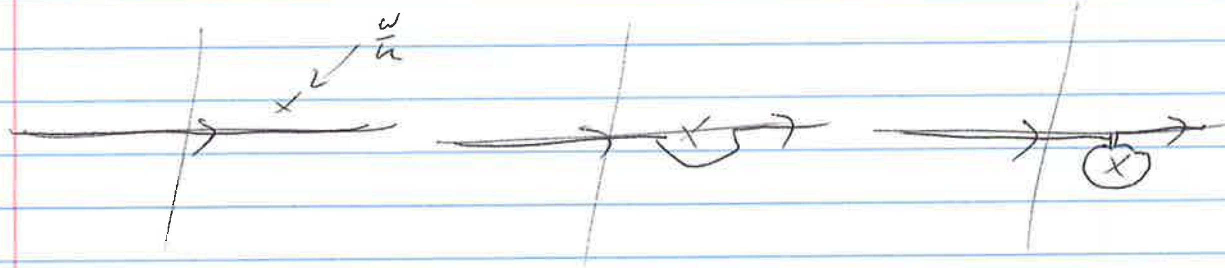
2018 Day 1 Question 4

a. Landau contour has three cases.

$\text{Im}\,\omega > 0$ $\qquad\qquad$ $\text{Im}\,\omega = 0$ $\qquad\qquad$ $\text{Im}\,\omega < 0$



b. Suppose there is an instability, $\text{Im}\,\omega > 0$

$$1 + \frac{\omega_p^2}{k^2}\int \frac{g'}{\frac{\omega}{k} - v}\,dv = 0$$

$$1 + \frac{\omega_p^2}{k^2}\int \frac{\frac{k}{\omega}\left(\frac{\omega}{k} - v + v\right)g'}{\frac{\omega}{k} - v}\,dv = 1 + \frac{k}{\omega}\frac{\omega_p^2}{k^2}\int g' + \frac{vg'}{\frac{\omega}{k} - v}\,dv$$

note $\int g'\,dv = g\Big|_{-\infty}^{\infty} = 0.$

$$= 1 + \frac{k}{\omega}\frac{\omega_p^2}{k^2}\int \frac{vg'}{\frac{\omega}{k} - v}\,dv = 1 + \frac{k}{\omega}\frac{\omega_p^2}{k^2}\int \frac{\left(\frac{\omega^*}{k} - v\right)vg'}{\left|\frac{\omega}{k} - v\right|^2}\,dv$$

~~Imaginary part gives~~ $\cancel{\text{Im}\,\omega\, k\frac{\omega_p^2}{k^2}\int \frac{\left(\frac{\text{Im}\,\omega}{k} - v\right)vg'}{\left|\frac{\omega}{k} - v\right|^2} = 0.}$

~~By assumption,~~ $\cancel{vg' < 0\ \forall v.}$ ~~And~~ $\cancel{\left|\frac{\omega}{k} - v\right|^2}$ ~~clearly positive.~~

$\cancel{\text{Im}\,\omega\,\Big(1 +}$

Imaginary part only: $\text{Im}\,\omega\left(1 - \frac{\omega_p^2}{k^2}\int \frac{vg'}{\left|\frac{\omega}{k} - v\right|^2}\,dv\right) = 0$

Since $vg' < 0$ by assumption (g monotonically decreasing function of v^2) and $\left|\frac{\omega}{k} - v\right|^2 > 0$, $\int \frac{vg'\,dv}{\left|\frac{\omega}{k} - v\right|^2} < 0$, hence

$$1 - \frac{\omega_p^2}{k^2}\int \frac{vg'}{\left|\frac{\omega}{k} - v\right|^2}\,dv \neq 0,$$

so we have contradicted that $\text{Im}\,\omega > 0 \longrightarrow$ the system must be stable.

Keep for reference for when you do this problem.

$$1 + \frac{\omega_p^2}{k^2}\int_L \frac{1}{\frac{\omega}{k} - v}\frac{\partial f_e}{\partial v}\,dv = 0$$

show $v\frac{\partial f}{\partial v} > 0 \Rightarrow \text{Im}\,\omega \overset{?}{\gtrless} 0.$

→ 2013 Day 2, Q1 (neoclassical)

$$\frac{V_{\parallel}}{V_{\perp}} < \sqrt{\frac{B_{max}}{B_{min}} - 1} \qquad B \simeq B_0\left(1 - \frac{r}{R}\cos\theta\right)$$

$$< \sqrt{2\epsilon} \qquad \rightarrow B_{max} = B_0(1+\epsilon)$$
$$B_{min} \quad B_0(1-\epsilon)$$

⟶ $f_{trapped} \sim \sqrt{2\epsilon}$

marginal trapping : $\frac{V_{\parallel}}{V_{\perp}} = \sqrt{2\epsilon} \rightarrow V_{\parallel} \sim V_{\perp}\sqrt{\epsilon}$

$P_{\phi}(R) = r m V_{\parallel} + R_{\xi} A(R)$ canonical angular momentum.

$$\Delta R \sim \frac{m V_{\parallel}}{e B_p} \sim \frac{q \rho_{\perp}}{\sqrt{\epsilon}}$$

$$J = e V_{th}\frac{\partial n}{\partial r}\Delta r \sim \frac{2\sqrt{\epsilon}}{\sqrt{B}\,B_{pT}}\frac{\partial P}{\partial r}$$