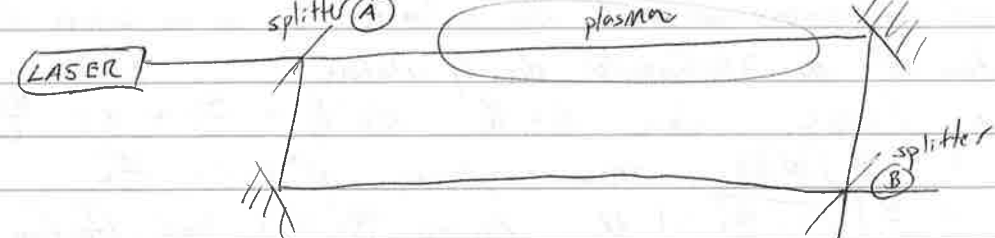
2013 Day 1 Question 2 (Diagnostics)

1. 

A single beam is split into two at point (A). Due to different index of refraction in the plasma vs air, the reference and plasma beam will have different phase when meeting at point B. In the plasma, $\omega^2 = \omega_p^2 + k^2c^2 \rightarrow N^2 = 1 - \frac{\omega_p^2}{\omega^2}$ so $N(n_e)$, hence a measurement of the electron density can be made.

Limitations: line-averaged density only. Must have knowledge of the path travelled, how much refraction, etc.

2a. Let $\Delta\phi = \Delta\phi^0 + \Delta\phi^1$ ($\Delta\phi$ is the total lab measurement)

Where $\Delta\phi^0$ is the unperturbed / vibrationless measurement $\Delta\phi^0 = K\lambda\int n\,d\ell$ and $\Delta\phi^1$ is the additional phase change due to the extra path length due to vibrations. If $\delta\ell$ is this additional path, then $\Delta\phi^1 = 2\pi\frac{\delta\ell}{\lambda}$.

$$\lambda_H(\Delta\phi_H - \Delta\phi_H^0) = 2\pi\delta\ell = \lambda_c(\Delta\phi_c - \Delta\phi_c^0)$$

$$\lambda_H\Delta\phi_H - \lambda_H^2 K\int n_e d\ell = \lambda_c\Delta\phi_c - \lambda_c^2 K\int n_e d\ell$$

$$(\lambda_c^2 - \lambda_H^2)K\int n_e d\ell = \lambda_c\Delta\phi_c - \lambda_H\Delta\phi_H$$

$$f = \int n_e d\ell = \frac{1}{K}\frac{\lambda_c\Delta\phi_c - \lambda_H\Delta\phi_H}{\lambda_c^2 - \lambda_H^2}$$

b. $$\delta f = \frac{1}{K}\frac{\delta(\lambda_c\Delta\phi_c) - \delta(\lambda_H\Delta\phi_H)}{\lambda_c^2 - \lambda_H^2}$$

$$= \frac{-\lambda_H\,\delta(\Delta\phi_H)}{K(\lambda_c^2 - \lambda_H^2)} = \frac{-\pi\lambda_H}{K(\lambda_c^2 - \lambda_H^2)}$$

$\rightarrow$

2. c. To get real numbers we must also evaluate K, so we include full derivation of the interferometer density relation.

$$\Delta\phi = \int K\,d\ell \qquad \text{where } v_p = \frac{\omega}{k},\ N = \frac{c}{v_p} = \frac{kc}{\omega} \Rightarrow k = \frac{N\omega}{c}$$

$$= \frac{\omega}{c}\int (N-1)\,d\ell \,. \quad \text{EM wave in plasma, } N^2 = 1 - \frac{\omega_p^2}{\omega^2}$$

$$= \frac{\omega}{c}\int \sqrt{1-\frac{\omega_p^2}{\omega^2}} - 1\,d\ell \,. \quad \text{Assume } \frac{\omega_p^2}{\omega^2} \ll 1 \text{ limit (far from cutoff)}$$

$$\approx -\frac{\omega}{2c}\int \frac{\omega_p^2}{\omega^2}\,d\ell = \frac{\omega}{2c}\int \frac{4\pi n e^2}{m\omega^2}\,d\ell = \frac{e^2}{mc^2}\lambda \int n\,d\ell$$

computer-assisted

$$\text{So } K = \frac{e^2}{mc^2} = \frac{(4.8\text{E-}10 \text{ statC})^2}{(9.1\text{E-}28\text{ g})(3\text{E}10 \text{ cm/s})^2} \approx \frac{25\text{E-}20}{27\text{E-}8} \approx 10^{-14}\text{ cm}$$

bad algebra/estimation?

In a 10^{14} cm^{-3} plasma 1 m thick,

$$f = \int n_e\,d\ell \approx 10^{16}\text{ cm}^{-2}$$

$$\text{So } \frac{\delta f}{f} = \frac{1}{Kf}\,\frac{\pi(\lambda_H)}{\lambda_H^2 - \lambda_c^2} \cong \frac{\pi(.6\text{E+}4\text{ cm})}{100\text{ cm}^{-1}(10.6\text{E+}4)^2} \approx$$