2013 Pay1 Qnsh 5B (draft) (computational)

a. $\frac{\partial B}{\partial t} = -\nabla \times \left(\alpha (\nabla \times B) \times B \right)$ $\qquad B = B_x(z)\hat{x} + B_y(z)\hat{y} + B_z\hat{z}$, $\quad B_z = B_0$

$$\nabla \times B = \begin{matrix} \partial_y B_z - \partial_z B_y \\ \partial_z B_x - \partial_x B_z \\ \partial_x B_y - \partial_y B_x \end{matrix} = \begin{pmatrix} -B_y' \\ B_x' \\ 0 \end{pmatrix} = A$$

$$\underbrace{(\nabla \times B)}_{A} \times B = \begin{matrix} A_y B_z - A_z B_y \\ A_z B_x - A_x B_z \\ A_x B_y - A_y B_x \end{matrix} = \begin{pmatrix} B_x' B_0 \\ + B_y' B_0 \\ - B_y' B_y - B_x' B_x \end{pmatrix}$$

$$\nabla \times \left((\nabla \times B) \times B \right) = \begin{pmatrix} - B_0 B_y'' \\ B_0 B_x'' \\ 0 \end{pmatrix}$$

$$\frac{\partial B}{\partial t} = \alpha \begin{pmatrix} B_0 B_y'' \\ - B_0 B_x'' \end{pmatrix} = \alpha B_0 \begin{pmatrix} & 1 \\ -1 & \end{pmatrix} B''$$

$$\cancel{\frac{B_{n+1} - B_n}{\Delta t}} = \cancel{B_{n+1} - B_n} \; \frac{\alpha B_0}{\Delta x^2} \Big($$

$$\frac{B_i^{n+1} - B_i^n}{\Delta t} = \frac{\alpha B_0}{\Delta x^2} \left(U_{i+1}^n - 2U_i^n + U_{i-1}^n \right)$$

b. stability analysis: error also obeys this equation.
Fourier Analysis: $\epsilon \sim e^{at} e^{ik_m x}$

$$G = \frac{\epsilon_j^{n+1}}{\epsilon_j^n} \leq 1 \quad \text{for stability.}$$

c. improved methods: Dufort-Frankel: CTCS, with $U_i^n = \frac{1}{2}\left(U_i^{n+1} + U_i^{n-1}\right)$
unconditionally stable in time

(or BTCS) $\rightarrow$ same as FTCS except evaluate spatial derivative terms at forward time