

a. $\frac{\partial B}{\partial t} = -\nabla \times (\alpha (\nabla \times B) \times B)$

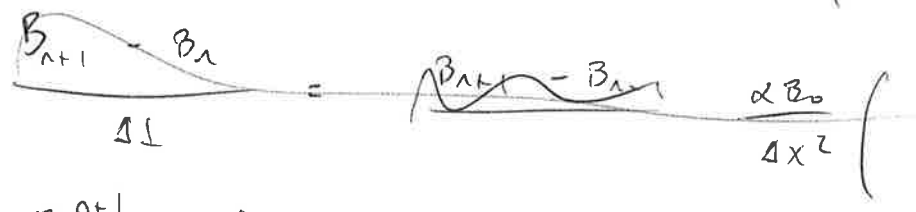
$B = B_x(z) \hat{x} + B_y(z) \hat{y} + B_z(z) \hat{z}$

$\nabla \times B = \begin{pmatrix} \partial_y B_z - \partial_z B_y \\ \partial_z B_x - \partial_x B_z \\ \partial_x B_y - \partial_y B_x \end{pmatrix} = A$

$(\nabla \times B) \times B = \begin{pmatrix} A_y B_z - A_z B_y \\ A_z B_x - A_x B_z \\ A_x B_y - A_y B_x \end{pmatrix} = \begin{pmatrix} B_x' B_0 \\ + B_y' B_0 \\ - B_y' B_y - B_x' B_x \end{pmatrix}$

$\nabla \times ((\nabla \times B) \times B) = \begin{pmatrix} -B_0 B_y'' \\ B_0 B_x'' \\ 0 \end{pmatrix}$

$\frac{\partial B}{\partial t} = \alpha \begin{pmatrix} B_0 B_y'' \\ -B_0 B_x'' \end{pmatrix} = \alpha B_0 \begin{pmatrix} 1 \\ -1 \end{pmatrix} B''$



$\frac{B_i^{n+1} - B_i^n}{\Delta t} = \frac{\alpha B_0}{\Delta x^2} (U_{i+1}^n - 2U_i^n + U_{i-1}^n)$

b. stability analysis: error also obeys this equation.

Fourier Analysis: $E \sim e^{at} e^{ik_m x}$

$G = \frac{E_j^{n+1}}{E_j^n} \leq 1$ for stability.

c. improved methods: DuFort-Frankel: CTCS, with $U_i^n = \frac{1}{2}(U_i^{n+1} + U_i^{n-1})$ unconditionally stable in time

(w BTCS) \rightarrow same as FTCS except evaluate spatial derivative terms at forward time