2013 Day 1 Question 6 (GPP)

$\mu B_{max} < W$ trapping condition

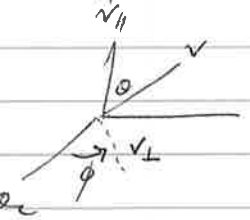
$W_\perp \dfrac{B_{max}}{B_0} < W$ ($\mu B_0 = W_\perp$ at midplane)

Define $R = B_{max}/B_0$

$W_\perp R < W_\perp + W_\parallel$

$W_\perp (R-1) < W_\parallel \rightarrow V_\perp < (R-1)^{-1/2} V_\parallel$

$V_\perp = (R-1)^{-1/2} V_\parallel$



$$f_{trapped} = \frac{\int_{\theta_c}^{\pi-\theta_c} r^2 \sin\theta \, d\theta \, d\phi \, dr}{\int r^2 \sin\theta \, d\theta \, d\phi \, dr} = \frac{\int_{\theta_c}^{\pi-\theta_c} \sin\theta \, d\theta}{2}$$

$= \frac{1}{2}\left(-\cos(\pi-\theta_c) + \cos\theta_c\right) = \cos\theta_c .$

θ_c defined by $\tan\theta_c = \dfrac{V_\perp}{V_\parallel} \rightarrow \cos\theta_c = \dfrac{V_\parallel}{V}$

From above, $W_\perp R < W$

$(W - W_\parallel) R < W$

$W \dfrac{(R-1)}{R} < W_\parallel \rightarrow \dfrac{V_\parallel}{V} = \sqrt{1 - R^{-1}} = f.$