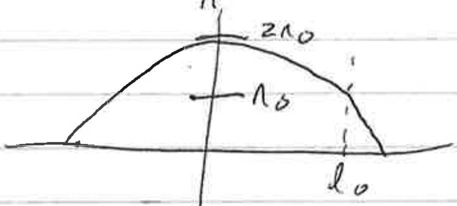
2015 Day 1 Q7 Waves

Assume $E \sim e^{ikx}$, we want to calculate the change in phase of the wave across the evanescent region.

This is an O-Mode. $\omega^2 = \omega_p^2 + k^2c^2$.

$n(x) = n_0\left(2 - \frac{x^2}{\ell_0^2}\right)$ n_0 cutoff :



(WKB - like approximation)

So $\Delta\phi = 2\int_0^{\ell_0} k \cdot d\ell = \frac{2i}{c}\int_0^{\ell_0}\sqrt{\omega_p^2 - \omega^2}\,dx$

$= \frac{2i}{c}\sqrt{\frac{4\pi n_0 e^2}{m}}\int_0^{\ell_0}\sqrt{\left(2 - \frac{x^2}{\ell_0^2}\right) - 1}\,dx$

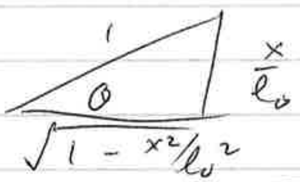
Compute $\int_0^{\ell_0}\sqrt{1 - x^2/\ell_0^2}\,dx$

$= \ell_0\int_0^{\frac{\pi}{2}}\cos^3\theta\,d\theta$

$= \ell_0\sin\theta\Big|_0^{\pi/2} - \ell_0\int_0^{\pi/2}\sin^2\theta\cos\theta$

$- \ell_0\frac{1}{3}\sin^3\theta\Big|_0^{\pi/2}$

$= \frac{2}{3}\ell_0$.



$\cos\theta = \sqrt{1 - x^2/\ell_0^2}$

$-\sin\theta\,d\theta = -\frac{x\,dx}{\ell_0^2}\frac{1}{\cos\theta}$

$\ell_0\frac{\sin\theta\cos\theta}{\tan\theta}\,d\theta = dx$

$\longrightarrow \Delta\phi = i\frac{4}{3c}\omega_p\ell_0$

So $\log\tau = |\Delta\phi| = \frac{4}{3c}\omega_p\ell_0$.