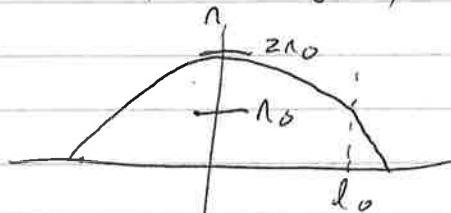


2015 Day 1 Q7 Wnvis

Assume $E \sim e^{ikx}$ we want to calculate the change in phase of the wave across the evanescent region.

This is an O-Mode. $\omega^2 = w_p^2 + k^2 c^2$.

$$n(x) = n_0 \left(1 - \frac{x^2}{l_0^2} \right) \quad n_0 \text{ cutoff:}$$



(WKB-like approximation)

$$\begin{aligned} \text{So } \Delta\phi &= \frac{e}{c} \int_0^{l_0} k \cdot dl = \frac{e}{c} \int_0^{l_0} \sqrt{w_p^2 - \omega^2} dx \\ &= \frac{e}{c} \sqrt{\frac{4\pi n_0 e^2}{m}} \int_0^{l_0} \sqrt{\left(1 - \frac{x^2}{l_0^2}\right)} dx \end{aligned}$$

$$\text{Compute } \int_0^{l_0} \sqrt{1 - \frac{x^2}{l_0^2}} dx$$

$$\begin{aligned} &= l_0 \int_0^{\frac{\pi}{2}} \cos^2 \theta d\theta \\ &= l_0 \sin \theta \Big|_0^{\frac{\pi}{2}} - l_0 \int_0^{\frac{\pi}{2}} \sin^2 \theta \cos \theta d\theta \\ &\quad - l_0 \frac{1}{3} \sin^3 \theta \Big|_0^{\frac{\pi}{2}} \\ &= \frac{2}{3} l_0. \end{aligned}$$

$$\begin{aligned} &\text{Diagram: A right-angled triangle with hypotenuse } \sqrt{1 - \frac{x^2}{l_0^2}}, \text{ angle } \theta \text{ at bottom-left vertex.} \\ &\cos \theta = \frac{x}{l_0} \quad \frac{x}{l_0} \cos \theta \\ &-\sin \theta dx = -\frac{x dx}{l_0^2} \quad \frac{1}{l_0^2} \cos \theta \end{aligned}$$

$$\rightarrow \Delta\phi = \frac{i}{2} \frac{w_p l_0}{3}$$

$$\frac{i w_p l_0}{3} \int_0^{l_0} \frac{\sin \theta \cos \theta d\theta}{\cos \theta} = dx$$

$$\text{So } \log \approx = |\Delta\phi| = \frac{4}{3} \frac{w_p l_0}{3}$$