2010 II : Q1 Waves

1.) Electrostatic waves have $\vec{E} = -\nabla\phi \Rightarrow \vec{E} \parallel \vec{k}$

It is sufficient to have $N^2 >> \epsilon_{i,j}$

It is necessary but not sufficient to have a resonance condition ($N \to \infty$).

$$n_\perp^2 \left(\frac{\omega_{pi}^2}{\Omega_i^2 - \omega^2} \right) + n_{\parallel}^2 \frac{2\omega_{pe}^2}{k_{\parallel}^2 v_{te}^2} = 0 \qquad \frac{\omega_{pe}^2}{\omega_{pi}^2} = \frac{m_i}{Z_i m_e}$$

$$n_\perp^2 = -\frac{c^2 k_{\parallel}^2}{\omega^2} \left(\frac{2\omega_{pe}^2}{k_{\parallel}^2 v_{te}^2} \right) \left(\frac{\Omega_i^2 - \omega^2}{\omega_{pi}^2} \right) \qquad k_{\parallel}^2 v_{te}^2 >> 2\omega^2$$

$\Rightarrow n_{\parallel}^2$ large but const.

$$= + \frac{2c^2}{v_{te}^2} \left(\frac{\omega_{pe}^2}{\omega_{pi}^2} \right) \left(1 - \frac{\Omega_i^2}{\omega^2} \right)$$

$\Rightarrow$ no choice of ω can make $n_\perp^2 = \frac{c^2 k_\perp^2}{\omega^2} \to \infty$

$n_{\parallel}^2$ will never be infinite because the dispersion relation is independent of $k_{\parallel}$. Thus no resonances exist.

∴ The wave is electromagnetic.

2) see part (1). No resonances exist.

3) $N \to 0$ is a cutoff. In this case $k_\perp \to 0$.

$n_\perp^2 \to 0$ if $\omega \approx \Omega_i$ so cutoff at $\omega = \Omega_i$.