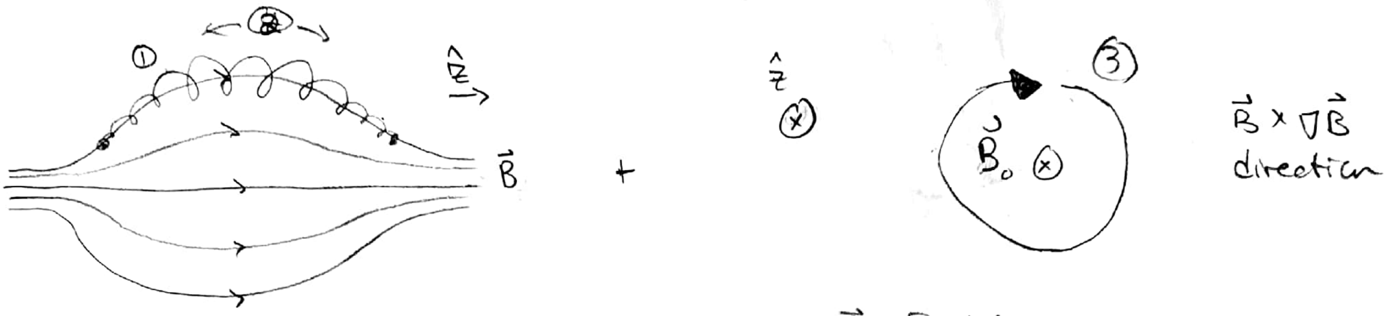
2010 II: Q3A GPP

a.)



THREE motions: (1) gyroorbit around $\vec{B}$-field.

(2) bounce between turning points.

(3) drift orbit around device.

b.) $v_{\parallel}(z=0) \equiv u$, $v_{\perp}(z=0) \equiv v$ $\qquad W_{\perp 0} = \frac{1}{2} m v^2 = \mu B_0$

energy conservation: $W_{\parallel} + W_{\perp} = W_{\parallel 0} + W_{\perp 0} \Rightarrow W_{\parallel} = W_{\parallel 0} + W_{\perp 0}\left(1 - \frac{B(z)}{B_0}\right)$

$$W_{\parallel} = W_{\parallel 0} + W_{\perp 0}\left(1 - \left[\frac{r_0^2}{r^2 + r_0^2}\right]^{2/3}\left(1 + 8\frac{z^2}{L^2}\right)\right)$$

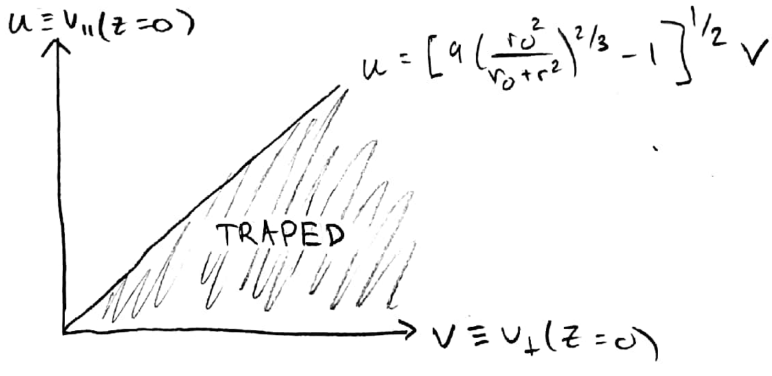
$$0 = W_{\parallel 0} + W_{\perp 0}\left(1 - \left[\frac{r_0^2}{r^2+r_0^2}\right]^{2/3}\left(1 + 8\frac{z_b^2}{L^2}\right)\right)$$

$$\Rightarrow z_b^2 = \left[\left(1 + \frac{W_{\parallel 0}}{W_{\perp 0}}\right)\left(\frac{r^2+r_0^2}{r_0^2}\right)^{2/3} - 1\right]\frac{L^2}{8}$$

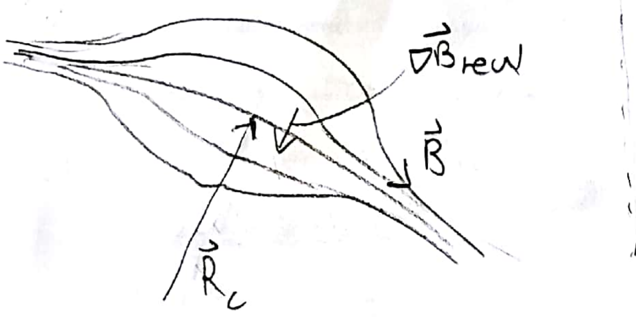
$$z_b^2 < L^2 \Rightarrow \left(1 + \frac{W_{\parallel 0}}{W_{\perp 0}}\right)\left(\frac{r^2+r_0^2}{r_0^2}\right)^{2/3} - 1 < 8 \qquad \frac{W_{\parallel 0}}{W_{\perp 0}} = \frac{u^2}{v^2}$$

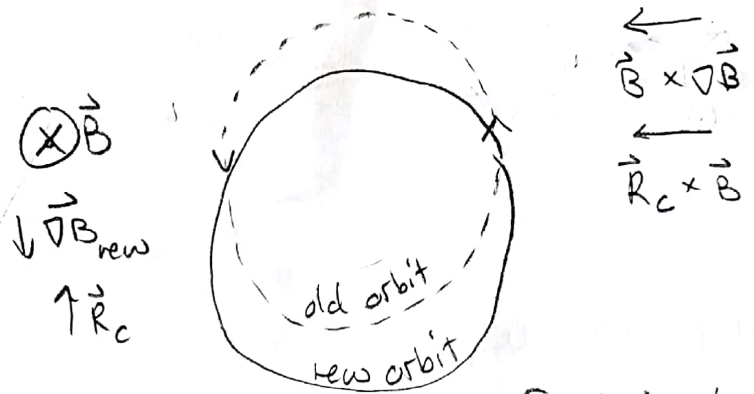
$$\Rightarrow \text{trapping requires} \quad \boxed{\frac{u^2}{v^2} < 9\left(\frac{r_0^2}{r^2+r_0^2}\right)^{2/3} - 1}$$

c.)



d.) Now bend the mirror with $R_c >> L$. Particles might still be trapped because if the extra $\vec{B} \times \nabla \vec{B}$ drift is slower than the drift orbit, the particles just get shifted radially. The curvature drift does the same thing





This is the same result as the poloidal field has in a tokamak. Of course it requires that the particles rotate radially faster than they drift out. At locations where the unbent curvature drift and the unbent ∇B drift cancel, this will no longer be the case - so particles that reach those locations along their bounce orbits will be lost.

e.) $v_{\nabla B} = \frac{m v_\perp^2}{2} \frac{\hat{B} \times \nabla \vec{B}}{q B^2}$, $V_{curv} = \frac{m v_\parallel^2}{R_c} \frac{\hat{R}_c \times \vec{B}}{q B^2}$

since particles are lost due to $v_{\nabla B}$ and v_{curv}, lighter particles are lost first.

Particles are untrapped when $\vec{V}_{\nabla B} = -\vec{V}_{curv}$

$$\frac{m v_\perp^2}{2B} \nabla \vec{B} \times \vec{B} = \frac{m u^2}{R_c} \hat{R}_c \times \vec{B} \Rightarrow \frac{u^2}{V^2} \sim \frac{R_c}{2B} \frac{\nabla \vec{B} \times \vec{B}}{\hat{R}_c \times \vec{B}}$$

I'm sure you can get a better answer by comparing drift speeds and calculating R_c, but I don't have time.

f.) If the machine charges positive you will generate a radial electric field which will create an azimuthal $E \times B$ drift and increase the rotational transform - thus trapping more particles of both species.