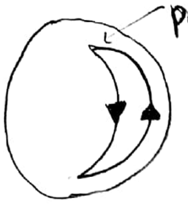
2010 I: Q4 Transport

a.) Banana orbit crossection:



particle mirrors on high-field side

particle also rotates toroidially

3D:

b.) $P_\zeta = mRv_\zeta + \frac{e}{c}\psi$

At the turning point ① $P_\zeta = \frac{e}{c}\psi(r)$

At the midplane, $(r, \theta) = (r+\Delta, 0)$

$$\Rightarrow ② \; P_\zeta = m(R_0 + r + \Delta)v_\zeta + \frac{e}{c}\psi(r+\Delta)$$

but $\psi(r+\Delta) \approx \psi(r) + \Delta\hat{r}\cdot\nabla\psi = \psi(r) + \Delta(R_0+r)B_p$

$$① = ② \Rightarrow \frac{e}{c}\cancel{\psi(r)} = m(R_0 + r + \Delta)v_\zeta + \frac{e}{c}\cancel{\psi(r)} + \frac{e}{c}\Delta(R_0+r)B_p$$

$$\Delta = \frac{m(\cancel{R_0 + r + \Delta})v_\zeta}{\frac{e}{c}(\cancel{R_0+r})B_p} \approx \frac{mv_\zeta}{\frac{e}{c}B_p}$$

Note that $v_\zeta \approx v_{th}\epsilon^{1/2}$ and $q = \epsilon\frac{B_T}{B_p}$ so

$$\Delta = \frac{mv_{th}}{e/cB_T}\frac{q}{\epsilon^{1/2}} \Rightarrow \boxed{\Delta \sim \rho q e^{-1/2}}$$

c.) $D_{banana} = f_{trapped}\frac{(\Delta x)^2}{(\Delta t)} = f_T\Delta^2\nu_{eff}$ $\qquad \nu_{eff} \sim \frac{\nu_{ei}^{90°}}{\epsilon}$ $\qquad f_T \sim e^{1/2}$

$$\Rightarrow \boxed{D_{banana} \sim \rho^2 q^2 \epsilon^{-3/2}\nu_{ei}^{90°}}$$

(d.) $D_{PS} = \frac{(\Delta x_{PS})^2}{(\Delta t)} = (v_d)^2\Delta t_{\parallel}$

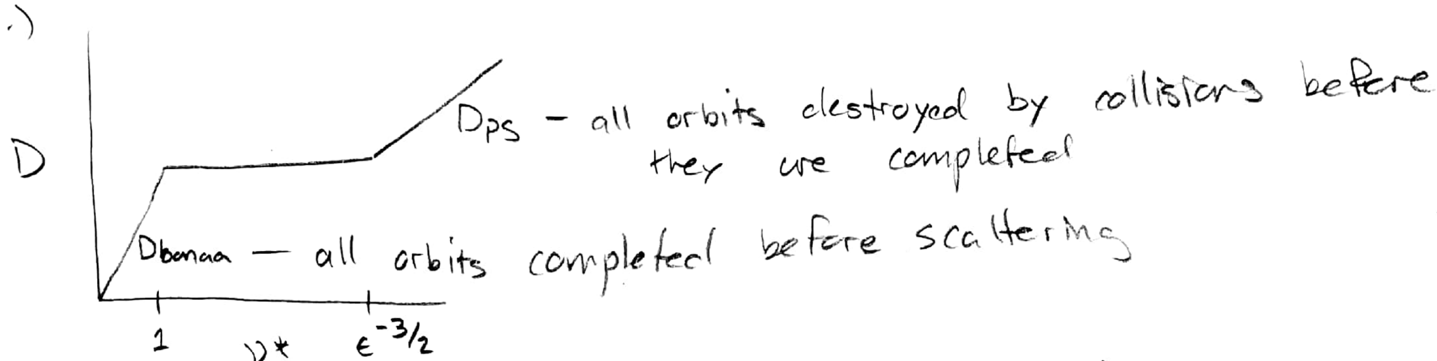
$$v_d \sim \frac{1}{\Omega}(\hat{r}\times\mu\nabla B) \sim \frac{\mu}{\Omega}\frac{B}{R_0} \sim \frac{v_{th}^2}{\Omega\cancel{B}}\frac{\cancel{B}}{R_0} \sim \frac{\rho v_{th}}{R_0}$$

We get $\Delta t_{\parallel}$ from $D_{\parallel} \sim \frac{(\Delta x_{\parallel})^2}{\Delta t_{\parallel}} \sim \frac{(R_0 q)^2}{\Delta t_{\parallel}} \sim \frac{v_{th}^2}{\nu_{ei}^{90^\circ}}$

$$\Rightarrow \Delta t_{\parallel} \sim \nu_{ei}^{90^\circ} (R_0 q)^2 \frac{1}{v_{th}^2}$$

Then $D_{PS} = \left(\frac{\rho v_{th}}{R_0}\right)^2 \frac{1}{v_{th}^2} \nu_{ei}^{90^\circ} (R_0 q)^2 \Rightarrow \boxed{D_{PS} \sim \rho^2 q^2 \nu_{ei}^{90^\circ}}$

(e.)



Note: $\nu^* = \frac{\nu_{eff}}{\omega_b} \sim \frac{\nu_{ei}^{90^\circ}}{\epsilon}\left(\frac{R_0 q}{v_{th}\, \epsilon^{1/2}}\right) \sim \frac{\nu_{ei}^{90^\circ} R_0 q}{v_{th}} \epsilon^{-3/2}$

$D_{banana}: \nu^* < 1, \qquad D_{PS}: \nu^* > \epsilon^{-3/2}$

(e.) $\lim_{\nu^* \to 1} D_{banana} = \lim_{\nu^* \to 1} \left(\rho^2 q^2 \underbrace{\left[\epsilon^{-3/2} \nu_{ei}^{90^\circ}\left(\frac{R_0 q}{v_{th}}\right)\right]}_{1}\left(\frac{v_{th}}{R_0 q}\right)\right)$

$$= \frac{\rho^2 q v_{th}}{R_0}$$

$\lim_{\nu^* \to \epsilon^{-3/2}} D_{PS} = \lim_{\nu^* \to \epsilon^{-3/2}} \left(\rho^2 q^2 \underbrace{\left[\nu_{ei}^{90^\circ}\left(\frac{R_0 q\, \epsilon^{-3/2}}{v_{th}}\right)\right]}_{\cancel{\epsilon^{-3/2}}}\left(\frac{v_{th}}{R_0 q\, \cancel{\epsilon^{-3/2}}}\right)\right)$

$$= \frac{\rho^2 q v_{th}}{R_0}$$

So $\boxed{D_{plateau} \sim \frac{\rho^2 q v_{th}}{R_0}}$