2011 II : 2 Waves

a.) Use cold magnetized plasma dispersion relation.

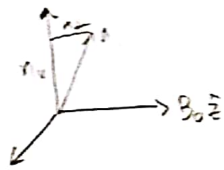
$$\epsilon = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix} \quad \text{where} \quad S = \tfrac{1}{2}(R+L), \quad D = \tfrac{1}{2}(R-L)$$

$$R = 1 - \sum_s \frac{\omega_{ps}^2}{\omega(\omega+\Omega_s)} \qquad P = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2}$$

$$L = 1 - \sum_s \frac{\omega_{ps}^2}{\omega(\omega-\Omega_s)}$$

$$D = N_i N_j - N^2 \delta_{ij} + \epsilon_{ij}$$

$$\Rightarrow D = \begin{pmatrix} S - N_\parallel^2 & -iD & n_\perp n_\parallel \\ iD & S - N^2 & 0 \\ n_\perp n_\parallel & 0 & P - N_\perp^2 \end{pmatrix} \qquad (n_x = n_\parallel,\ n_y = 0,\ n_z = n_\perp)$$



X-mode has $n_\parallel = 0$ so $S(S-N^2) - D^2 = 0$

$$N^2 = \frac{S^2 - D^2}{S} = \frac{1}{S}\left[\tfrac{1}{4}(R^2+2RL+L^2) - \tfrac{1}{4}(R^2-2RL+L^2)\right] = \frac{RL}{S} \checkmark$$

Resonance is $N^2 \to \infty$, so $S \to 0$

$$\Rightarrow \frac{1}{2}\left[1 - \frac{\omega_{pe}^2}{\omega(\omega+\Omega_e)} - \frac{\omega_{pi}^2}{\omega(\omega+\Omega_i)} + 1 - \frac{\omega_{pe}^2}{\omega(\omega-\Omega_e)} - \frac{\omega_{pi}^2}{\omega(\omega-\Omega_i)}\right] = 0$$

$$1 - \frac{\omega_{pe}^2}{\omega^2 - \Omega_e^2} - \frac{\omega_{pi}^2}{\omega^2 - \Omega_i^2} = 0$$

For upper hybrid take $\omega \gg \Omega_i, \omega_{pi}$

$$\Rightarrow \boxed{\omega_{UH}^2 = \omega_{pe}^2 + \Omega_e^2}$$

b.) Electrostatic $\Rightarrow \vec{E} \parallel \vec{k}$

$$\begin{pmatrix} S & -iD \\ iD & S - N^2 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \end{pmatrix} = 0 \quad \Rightarrow iD E_x + (S - N^2) E_y = 0$$

$$\Rightarrow \frac{E_x}{E_y} = \frac{N^2 - S}{iD} \gg 1 \quad \text{since } N^2 \gg S,\ N^2 \gg iD$$

$\hookrightarrow$ thus $\vec{E} \parallel \vec{k} \Rightarrow$ electrostatic.