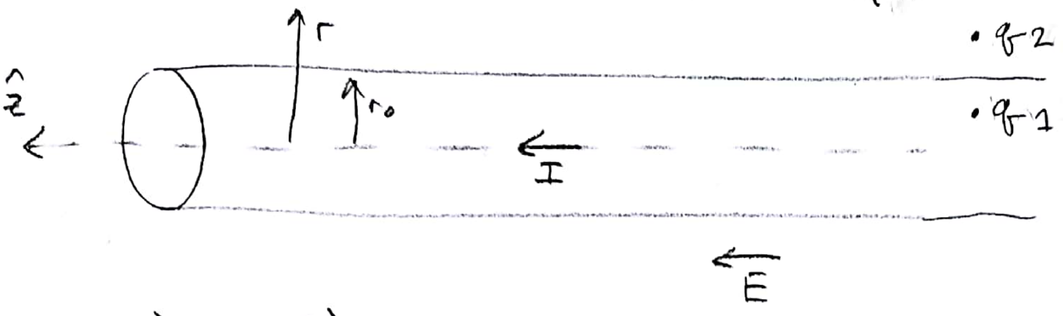
2011 I: Q 1 A Quickie



$\Big\}\ \rho_i(r_0) \ll r_0$

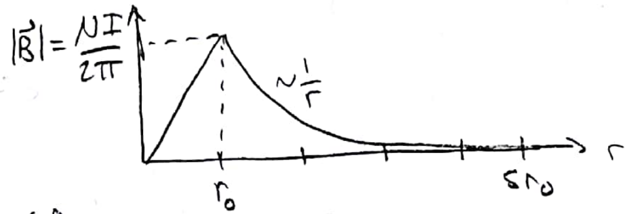
⇓

guiding centers!

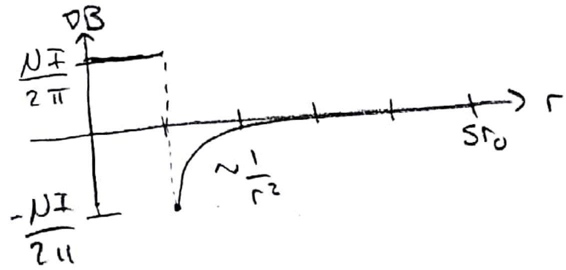
a.) $\nabla \times \vec{B} = \mu \vec{J}$

$$\int \nabla\times\vec{B}\, dA = \oint \vec{B}\, ds = 2\pi r B = \mu \int_0^r \vec{J}\, d\vec{A} = \mu I_{enc} = \mu \begin{cases} \frac{\pi r^2}{\pi r_0^2} I & r < r_0 \\ I & r > r_0 \end{cases}$$

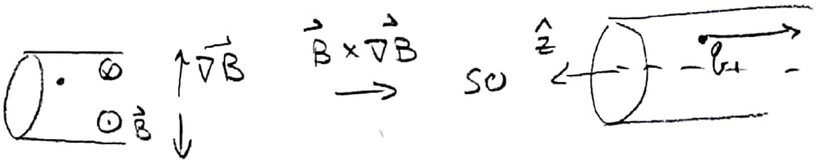
$$\Rightarrow B = \frac{\mu I}{2\pi}\begin{cases} r, & r < r_0 \\ 1/r, & r > r_0 \end{cases} \Rightarrow$$



$$\nabla B = \frac{\partial B}{\partial r} = \frac{\mu I}{2\pi}\begin{cases} \frac{\mu I}{2\pi} & r < r_0 \\ -\frac{\mu I}{2\pi r^2} & r > r_0 \end{cases} \Rightarrow$$

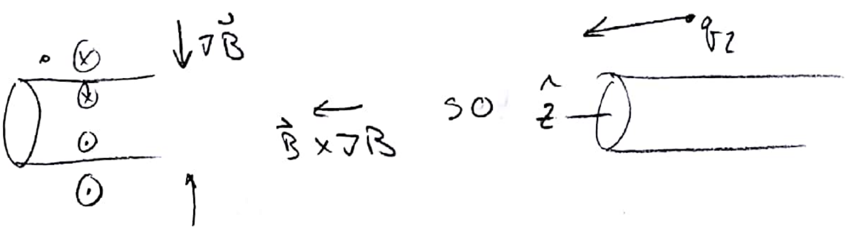


b.) q_1 has $r < r_0$



$\rho \ll r_0$

q_2 has $r > r_0$



$\Big\}\ \vec{E} = 0$

c.) Now let $E > 0$, $E \ll v_{i0} B(r_0)$

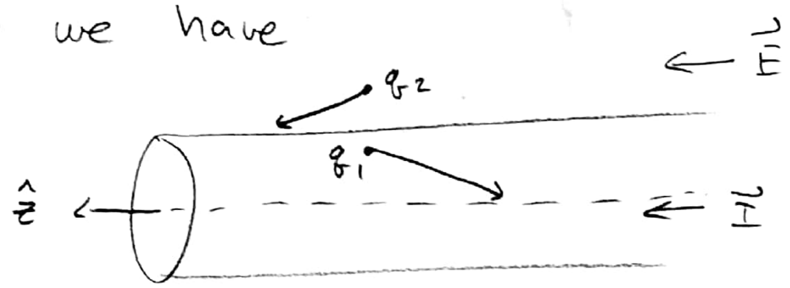
For both particles, the $\vec{E}\times\vec{B}$ drift is in $-\hat{r}$ direction

$$\vec{v}_{E\times B} = \frac{\vec{E}\times\vec{B}}{B^2} \qquad \vec{v}_{\nabla B} = W_{\perp}\frac{2\,\vec{B}\times\nabla B}{q B^2}$$

$$\frac{\cancel{q} E}{\cancel{q} v B} \ll 1$$

so guiding center drifts work.

so we have



d.) A particle initially at rest at $r=0$ will only feel $\vec{E} = E\hat{z}$ because $\vec{B}(r=0)=0$. A particle with a small initial velocity V_r will be push back to $r=0$ via the $\vec{E}\times\vec{B}$ drift. So if the geometry remains constant both particles should reach $r=0$.