2011 I : Q3 Diagnostics

a.) For nonrelativistic electrons we can use the dipole approximation for the scattered field. So $\vec{E}_s = \frac{r_e}{R} \hat{s} \times \hat{s} \times \vec{E}_i$, where $\vec{E}_i$ is the incident wave $\left(\dot{\vec{v}} = -\frac{e}{m} \vec{E}_i\right)$. Then $\frac{dP}{d\Omega_s} = R^2 c \varepsilon_0 |E_s|^2$

$$\frac{dP}{d\Omega_s} = r_e^2 c \varepsilon_0 \sin^2\phi \, |E_i|^2 \qquad \frac{d\sigma}{d\Omega_s} = \frac{dP/d\Omega_s}{c\varepsilon_0 |E_i|^2} \begin{array}{l} \leftarrow \text{scatter power} \\ \leftarrow \text{incident power} \end{array}$$

$$\Rightarrow \frac{d\sigma}{d\Omega_s} = r_e^2 \sin^2\phi \qquad \sigma = \int \frac{d\sigma}{d\Omega_s} d\Omega_s = \int \frac{d\sigma}{d\Omega_s} 2\pi \sin\phi \, d\phi$$

$$\sigma = \int r_e^2 \, 2\pi \sin^3\phi \, d\phi = 2\pi r_e^2 \int_0^{\pi} d\phi \, \sin\phi\,(1-\cos^2\phi)$$

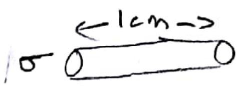
$$= 2\pi r_e^2 \left[-\cos\phi \Big|_0^{\pi} - \int_0^{\pi} \sin\phi \cos^2\phi \, d\phi \right] \qquad \begin{array}{l} u = \cos\phi \\ du = -\sin\phi \, d\phi \end{array}$$

$$= 2\pi r_e^2 \left[1 - \frac{1}{3} \cos^2\phi \Big|_0^{\pi} \right]$$

$$= 2\pi r_e^2 \left[1 + \frac{1}{3} \right]$$

$$\boxed{\sigma = \frac{8\pi}{3} r_e^2}$$

b.): Thomson scattering is from incident photons.



$$N_{scatt} = n_0 \sigma \Delta x \cdot N_{incoming}$$

Fraction scattered into 0.01 str $= \frac{N_{scat}}{N_{incoming}} = n_0 \sigma \Delta x \left(\frac{0.01}{4\pi}\right)$

$$f = (2\times 10^{20}) \frac{\cancel{8\pi}^{\,2}}{\cancel{3}} \cancel{(2.8\times10^{-15})}(2.8\times 10^{-15})(1\times10^{-2}) \frac{(1\times 10^{-2})}{\cancel{4\pi}}$$

$$\approx 4 \times 3 \times \underbrace{10^{20} \times 10^{-15} \times 10^{-15}}_{10^{-10}} \times 10^{-4}$$

$$\approx 12 \times 10^{-14}$$

$$\Rightarrow \boxed{f_{scatt} \sim 1 \times 10^{-13}}$$