2011 I: Q4 Transport

1a.) $$\Gamma_{neo} \sim -D_{banana}\left(\frac{dn_0}{dr} \underbrace{- \frac{n_0}{2}\frac{d\ln T_0}{dr}}_{\nabla T_0 \text{ contrib.?}}\right) \underbrace{- n_0\,\epsilon^{1/2}\frac{cE_\parallel}{B_p}}_{\text{Ware-pinch}}$$ 1

$(\Gamma = -D\nabla n)$

where we know $D_{banana} \sim \nu_{ei}\rho^2 q^2 \epsilon^{-3/2}$ ← see 2010I:Q4

1b.) $$\langle j_\parallel - j_s\rangle \sim \underbrace{-\epsilon^{1/2}\frac{c}{B_p}\frac{dP_e}{dr}}_{\text{Bootstrap current}} \underbrace{- \epsilon^{1/2}\sigma_\parallel E_\parallel}_{\substack{\text{neoclassical reduction of}\\ \text{classical conductivity}\\ \text{(since traped particles}\\ \text{carry no current)}}}$$ 2, 3 → memorize

with $P_e \equiv n_0 T_0 \Rightarrow \frac{dP_e}{dr} = T_0\frac{dn_0}{dr} + n_0\frac{dT_0}{dr}$

$$\begin{pmatrix} \Gamma_{neo} \\ \langle j_\parallel - j_s\rangle \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{D_{banana}}{T_0} & \epsilon^{1/2}\frac{c}{B_p} \\ \epsilon^{1/2}\frac{c}{B_p} & \frac{1}{n_0}\sigma_\parallel E_\parallel \end{pmatrix}\begin{pmatrix} \frac{dP}{dr} - \frac{5}{2}n_0\frac{dT_e}{dr} \\ -n_0 E_\parallel \end{pmatrix}}_{\text{need to check this}}$$

Derivation of Ware-pinch term:

1 $\Delta\theta \sim \frac{\Delta v_\parallel}{v_\parallel}$ and $\Delta v_\parallel \sim \frac{eE_\parallel}{m\omega_b} \sim \frac{eE_\parallel}{m}\frac{R_0 q}{v_\parallel} \sim \frac{eE_\parallel}{m}\frac{R_0 q}{v_{th}\epsilon^{1/2}}$

$$\Gamma_W \sim -\langle f_T\rangle n_0 v_W \sim -\epsilon^{1/2} n_0 (v_D\Delta\theta) \sim -\epsilon^{1/2} n_0\left(\frac{v_{th}\rho}{R_0}\right)\left(\frac{eE_\parallel}{m}\frac{R_0 q}{v_{th}\epsilon^{1/2}}\right)\left(\frac{1}{v_{th}\epsilon^{1/2}}\right)$$

$$\boxed{\Gamma_W \sim -n_0\,\epsilon^{1/2}\frac{cE_\parallel}{B_p}}$$

difference in density between co- and counter-moving particles

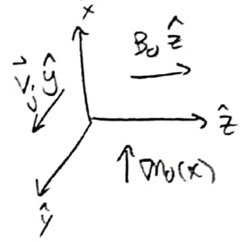
Bootstrap current: $j_b = ev_\parallel n_{c-c} \sim ev_\parallel \Lambda\frac{dn}{dr} \sim ev_\parallel\,\rho q\,\epsilon^{-1/2}\frac{dn}{dr}$

2 use $v_\parallel \sim v_{th}$ not $v_\parallel \sim v_{th}\epsilon^{1/2}$ b/c current carried by passing particles $\Rightarrow j_b \sim \frac{\epsilon^{1/2}}{B_p}cT\frac{dn}{dr} \rightarrow \boxed{j_b \sim \frac{c\epsilon^{1/2}}{B_p}\frac{dP_e}{dr}}$

2a.) Drift waves are a source of anomalous diffusion.

In the adiabatic limit, electrons tend to thermalize along field lines and follow the linearized Boltzmann distributions:

$$n_e = n_0 \exp\left(-\frac{e\phi}{kT_e}\right) \approx n_0\left(1+\frac{|e|\phi}{kT_e}\right) = n_{e0}+n_{e1}$$



For the "cold fluid" ions, use the fluid equations: $\Rightarrow$

$$\frac{\partial}{\partial t}(n_0+n_1)+\nabla\cdot\left[(n_0+n_1)(\vec{v}_0+\vec{v}_1)\right]=0 \qquad \vec{v}_0=0$$

$$\Rightarrow \frac{dn_1}{dt}+\nabla\cdot(n_0\vec{v}_1)=0 \Rightarrow \frac{\partial n_1}{\partial t}+\frac{\partial n_0}{\partial x}v_{1x}+n_0\nabla\cdot\vec{v}_1=0$$

$$m\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\vec{v}\right)=e(\vec{E}+\vec{v}\times\vec{B})-\frac{1}{n}\nabla p, \quad \vec{E}=-\nabla\phi, \quad P_i=0 \text{ since } T_i=0,$$
$$\phi=\phi(x)\exp[i(k_\perp y+k_\parallel z-\omega t)]$$

$$\Rightarrow \frac{\partial v_1}{\partial t}+\frac{e}{m}(\nabla\phi-\vec{v}_1\times\vec{B})=0$$

$$\vec{v}_1\times\vec{B}=\begin{vmatrix}\hat{x}&\hat{y}&\hat{z}\\ v_{1x}&v_{1y}&v_{1z}\\ & & B_0\end{vmatrix}$$

$$\hookrightarrow \begin{cases} +i\omega v_{1x}=+\frac{e}{m}(-v_{1y}B_0) \Rightarrow v_{1y}=-i\frac{\omega}{\Omega}v_{1x} \\ +i\omega v_{1y}=+\frac{e}{m}(v_{1x}B_0+ik_\perp\phi) \Rightarrow v_{1x}=i\frac{\omega}{\Omega}v_{1y}-i\frac{e}{m\Omega}k_\perp\phi \\ -\cancel{i}\omega v_{1z}=\cancel{+}\frac{e}{m}(\cancel{i}k_\parallel\phi) \Rightarrow v_{1z}=\frac{e}{m\omega}k_\parallel\phi \end{cases}$$

So $v_{1x}=i\frac{\omega}{\Omega}\left(-i\frac{\omega}{\Omega}\right)v_{1x}-i\frac{e}{m\Omega}k_\perp\phi \Rightarrow v_{1x}=\left(1-\cancelto{0}{\frac{\omega^2}{\Omega^2}}\right)^{-1}\left(-i\frac{e}{m\Omega}k_\perp\phi\right)$

Then the continuity eq. becomes: $-i\omega n_1+\frac{\partial n_0}{\partial x}\frac{e}{m\Omega}(-ik_\perp\phi)+n_0(\nabla\cdot v_1)\approx 0$

where $\nabla\cdot v_1=ik_\perp v_{1y}+ik_\parallel v_{1z}=i\left[-\frac{\omega}{\Omega}\frac{e}{m\Omega}k_\perp^2\phi+\frac{e}{m\omega}k_\parallel^2\phi\right]$

$$\Rightarrow n_1=n_0\left[\frac{e}{m}\frac{k_\parallel^2}{\omega^2}\phi-\frac{e}{m\Omega^2}k_\perp\phi-\frac{1}{n_0}\frac{dn_0}{dx}\frac{e}{m\Omega}\frac{k_\perp}{\omega}\phi\right]$$

enforce quasineutrality: $n_1e-n_1e=0 \Rightarrow \frac{e}{kT_e}=\frac{k_\parallel}{m\omega^2}-\frac{k_\perp}{m\Omega^2}-\frac{k_\perp}{m\Omega\omega}\frac{d\ln(n_0)}{dx}$

$$C_s\equiv\sqrt{\frac{kT}{m}} \Rightarrow \boxed{1=\frac{C_s^2k_\parallel}{\omega^2}-\frac{C_s^2k_\perp}{\Omega^2}\left(1+\frac{\Omega}{\omega}\frac{d\ln(n_0)}{dx}\right)}$$

2b.) Quasineutrality is justified because $\omega^{-1}\gg\omega_p^{-1}$?