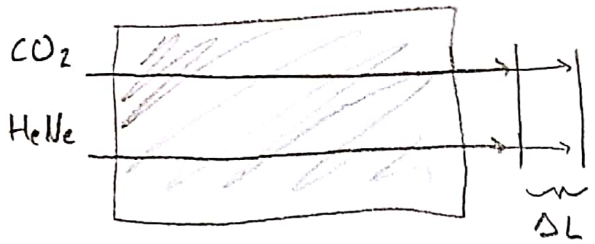
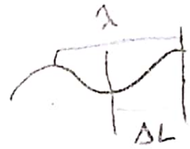
2012 II : Q5B Exp





a.) $\Delta\phi_{meas} = \Delta\phi_o + \Delta\phi_{vib}$ $\rightarrow$ $\Delta\phi_{vib} = 2\pi \frac{\Delta L}{\lambda}$

so $2\pi\Delta L = \lambda_H(\Delta\phi_m^H - \Delta\phi_o^H) = \lambda_c(\Delta\phi_m^c - \Delta\phi_o^c)$

we know $\Delta\phi_o = K\lambda \int N_e \, dl$ so

$$2\pi\Delta L = \lambda_H \phi_{He} - K\lambda_H^2 \int N_e dl = \lambda_c \phi_c - K\lambda_c^2 \int N_e dl$$

$$\Rightarrow \boxed{\int N_e dl = \frac{\lambda_H \phi_{He} - \lambda_c \phi_c}{K(\lambda_H^2 - \lambda_c^2)}}$$

b.) $$\delta\left(\int N_e dl\right) = \frac{\lambda_H(\delta\phi_{He}) - \lambda_c(\delta\phi_c)}{K(\lambda_H^2 - \lambda_c^2)}$$

suppose $\delta\phi_c \ll \delta\phi_{He}$ (since $\lambda_{He} \ll \lambda_c$) and $\delta\phi_{He} \sim \pi$

$$\boxed{\delta\left(\int N_e dl\right) \sim \frac{\pi}{K} \frac{\lambda_H}{\lambda_H^2 - \lambda_c^2}}$$

c.) $$\frac{\delta\left(\int N_e dl\right)}{\int N_e dl} \sim \frac{\pi}{k} \frac{\lambda_H}{\lambda_H^2 - \lambda_c^2} \frac{1}{(10^{20} m^{-2})}$$ $\int N_e dl = 10^{14} \frac{1}{cm^3} 10^6 \frac{cm^3}{m^3} 1\, m = 10^{20} cm^{-2}$

$$\frac{\lambda_H}{\lambda_H^2 - \lambda_c^2} \sim \frac{1 \times 10^{-6}}{(1-100) \times 10^{-12}} \left[\frac{1}{m}\right] \sim 10^{-2} \times 10^6 m \sim 10^4 m$$

Need to derive K ...

$$\Delta\phi = \int (k_{plasma} - k_o)\, dl = \int \frac{\omega}{c}(N-1)\, dl \qquad N = \frac{kc}{\omega},\ \mu_0 = 1$$

but $N = \frac{ck}{\omega}$ with $\omega^2 = \omega_p^2 + c^2k^2$ $\longrightarrow$ cutoff when $\omega^2 = \omega_p^2$

$$\omega^2 = \frac{4\pi n_c e^2}{m}$$

$$n_c = \frac{m\omega^2}{4\pi e^2}$$

$$\Rightarrow \frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2} = 1 - \frac{4\pi n e^2}{m\omega^2}$$

$$= 1 - \frac{n_e}{n_c}$$

$$\Rightarrow \Delta\phi = \frac{\omega}{c}\int \left(1 - \frac{n_e}{n_c}\right)^{1/2} - 1\ dl \qquad \omega >> \omega_p \Rightarrow n << n_c$$

$$\approx \frac{\omega}{c}\int \cancel{1} + \frac{n_e}{2n_c} - \cancel{1}\ dl$$

$$= \frac{\omega}{2cn_c}\int n_e\, dl = K\lambda \int N_e\, dl \quad \text{where } K = \frac{\omega/c\ 4\pi e^2}{2m\omega^2\ \lambda}$$

$$K = \frac{2\pi}{\omega}\frac{1}{\lambda}\frac{e^2}{mc} = \frac{e^2}{mc^2} \quad \underline{\text{in CGS}}$$

$$K \sim \frac{(5\times 10^{-10})^2}{9\times 10^{-28}\times 9\times 10^{20}} \sim 10^{-40+28}\ \text{cm} \sim 10^{-12}\ \text{cm} \sim 10^{-14}\ \text{m}$$

$$\Rightarrow \frac{\delta\int N_e\, dl}{\int N_e\, dl} \sim (10^{14})(10^4)(10^{-20}) \sim 1\%$$

d.) $\phi = K\lambda \int N_e$

$$\phi_c = (10^{-14})(10.6\times 10^{-6})(1)(10^{20}) \sim 10.6$$

$$\phi_H = (10^{-14})(0.6\times 10^{-6})(1)(10^{20}) \sim 0.6$$