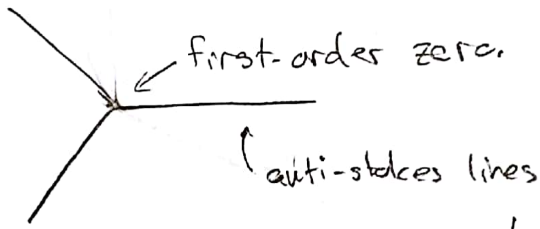
2012 I : 1B Math

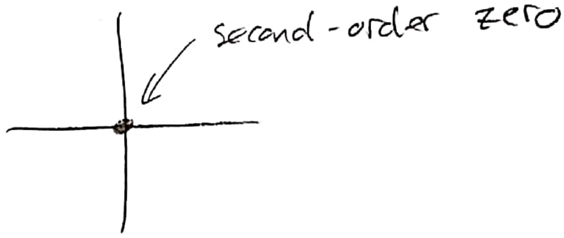
$$\frac{d^2y}{dx^2} + Q(z)\,y = 0 \qquad \text{WKBJ} \Rightarrow y_\pm \sim Q^{1/4} e^{\pm i\int \sqrt{Q(z)}\,dz}$$

Stokes and anti-stokes lines refer to the lines in the complex plane where the WKBJ solutions are purely exponential or oscillatory, respectively. For a n-th order turning point there are $n+2$ anti-stokes lines and $n+2$ stokes lines. So near a first order zero the lines are separated by $e^{\frac{2\pi}{3}i}$:

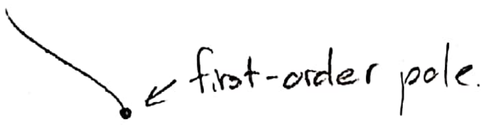
in general: rotated by $\arg(Q)/2$



For a second order zero they are separated by $e^{\frac{\pi}{2}i}$:



This rule also applies to $n=-1$ (first-order pole):



Derivation for First-order zero in $Q(z)$

Consider infinitesimal $t = z - z_0$ where z_0 is first-order zero.

$$\int Q^{1/2}dz \sim \int Q'(z_0)^{1/2} t^{1/2}\,dt \sim Q'(z_0)^{1/2} t^{3/2} \sim 1 \quad (e^{in\pi} \in \mathbb{R})$$

so $t \sim Q'(z_0)^{-1/3} e^{i(2\pi n/3)} \Rightarrow$ three lines.

for anti-stokes