2012 I: Q3 GPP

(a.) $W_{\perp 0} + W_{\parallel 0} = W_\perp + W_\parallel = \mu B + \frac{1}{2} m v^2$

$$W_\parallel = W_{\parallel 0} + \mu B_0 \left(1 - \left(1 + \frac{z^n}{L^n}\right)\right) = W_{\parallel 0} - W_{\perp 0}\frac{z^n}{L^n}$$

$$\Rightarrow z_T^n = L^n \left(W_{\parallel 0}/W_{\perp 0} \right)$$

But $z_T^n = c^n L^n$ so $z_T^n < (cL)^n$ is trapped

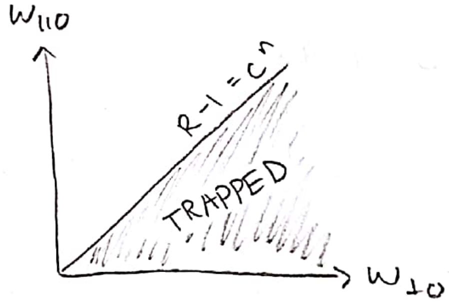
$$L^n\left(W_{\parallel 0}/W_{\perp 0}\right) < c^n L^n \Rightarrow \boxed{\frac{W_{\parallel 0}}{W_{\perp 0}} < c^n}$$

Easier: trapped if $\mu B_{max} > W_{\perp 0} + W_{\parallel 0}$

$$W_{\perp 0}\frac{B_{max}}{B_0} > W_{\perp 0} + W_{\parallel 0} \Rightarrow R > 1 + \frac{W_{\parallel 0}}{W_{\perp 0}}$$

$$R = 1 + c^n \Rightarrow \frac{W_{\parallel 0}}{W_{\perp 0}} < c^n$$

(b.)



$\frac{1}{2} m v_\parallel^2 =$

$\cancel{v_\parallel = \sqrt{\frac{2\varepsilon}{m}}}$

(c.) see part a. $\left(\frac{z_T}{L}\right)^n = \frac{W_{\parallel 0}}{W_{\perp 0}} \Rightarrow \boxed{\left(\frac{z_T}{L}\right)^2 = \frac{W_{\parallel 0}}{W_{\perp 0}}}$ for $n = 2$

(d.) $L \to \alpha L$, $B_0 \to \beta B_0$

μ conservation: $\mu = \frac{W_{\perp 0}}{B} \Rightarrow \frac{W_{\perp 0}}{\cancel{B_0}} = \frac{W'_{\perp 0}}{\beta \cancel{B_0}} \Rightarrow \boxed{W'_{\perp 0} = \beta W_{\perp 0}}$

(e.) $J = \oint m v_\parallel \, ds = \oint \sqrt{\frac{2 W_\parallel(z)}{m}} \, dz$

$$W_\parallel = W_{\parallel 0} - W_{\perp 0}\left(\frac{z}{L}\right)^n \Rightarrow J = 2\sqrt{\frac{2}{m}}\int_{-z_T}^{z_T} \sqrt{W_{\parallel 0}}\sqrt{1 - \frac{W_{\perp 0}}{W_{\parallel 0}}\left(\frac{z}{L}\right)^n} \, dz$$

let $u = \frac{W_{\perp 0}}{W_{\parallel 0}}\left(\frac{z}{L}\right)^n$ then $du = \frac{W_{\perp 0}}{W_{\parallel 0}}\frac{n}{L^n} z^{n-1} dz = u^{\frac{n-1}{n}}\left(\frac{W_{\perp 0}}{W_{\parallel 0}}\right)^{1/n}\frac{n}{L} dz$

$$J \sim \sqrt{W_{\parallel 0}}\int_{-1}^{1} du \; \frac{L}{n}\left(\frac{W_{\parallel 0}}{W_{\perp 0}}\right)^{1/n} \underbrace{u^{\frac{1-n}{n}}\sqrt{1-u}\, du}_{\text{same number...}}$$

$$\Rightarrow dz = \frac{L}{n}\left(\frac{W_{\parallel 0}}{W_{\perp 0}}\right)^{1/n} u^{\frac{1-n}{n}} du$$

$$J = J' \Rightarrow \cancel{L}\left(\frac{W_{\parallel 0}}{\cancel{W_{\perp 0}}}\right)^{1/n}(W_{\parallel 0})^{1/2} = \alpha \cancel{L}\,(W'_{\parallel 0})^{1/2}\left(\frac{W'_{\parallel 0}}{\beta \cancel{W_{\perp 0}}}\right)^{1/n}$$

$$\Rightarrow (w_{\parallel 0})^{\frac{1}{n}+\frac{1}{2}} = (w'_{\parallel 0})^{\frac{1}{n}+\frac{1}{2}} \quad \frac{\alpha}{\beta^{1/n}} \qquad \frac{1}{n}+\frac{1}{2} = \frac{2+n}{2n}$$

$$w'_{\parallel 0} = w_{\parallel 0}\left(\frac{\beta^{1/n}}{\alpha}\right)^{\frac{2n}{2+n}} \quad \Rightarrow \boxed{w'_{\parallel 0} = \left(\frac{\beta}{\alpha^n}\right)^{\frac{2}{2+n}} w_{\parallel 0}}$$

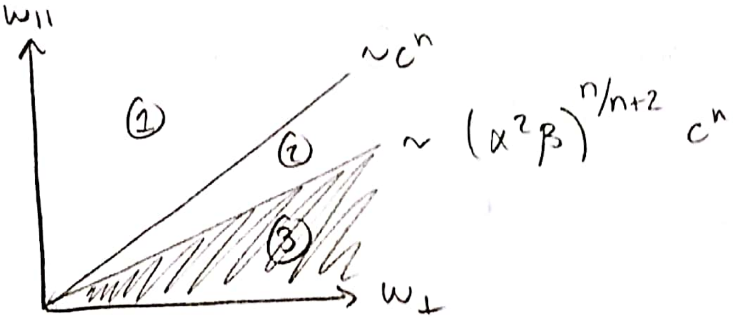
(f.) Initially trapped: $\frac{w_{\parallel 0}}{w_{\perp 0}} < c^n$ untrapped: $\frac{w'_{\parallel 0}}{w'_{\perp 0}} > c^n$

$$\left(\frac{\beta}{\alpha^n}\right)^{2/2+n} \frac{1}{\beta} \frac{w_{\parallel 0}}{w_{\perp 0}} > c^n$$

so $\frac{w_{\parallel 0}}{w_{\perp 0}} > \beta\left(\frac{\alpha^n}{\beta}\right)^{2/2+n} c^n$ become untrapped

for $n=2$ $\frac{w_{\parallel 0}}{w_{\perp 0}} > \alpha\beta^{1/2} c^n$ become untrapped.

(g.)



① - initially untrapped
② - now untrapped
③ - still trapped

(h.) Take $\beta < \alpha^{1/n}$ then $(\alpha^2\beta)^{n/n+2} < (\alpha^2\alpha^{1/n})^{n/n+2}$

$$= \left(\alpha^{\frac{2n+1}{n}}\right)^{\frac{n}{n+2}}$$

$$= \alpha^{\frac{2n+1}{n+2}}$$

$$< 1 \quad ??$$ not true in general...

So there are some newly untrapped particles.

* → should be $\alpha^{\frac{2n}{n+2}}\beta^{\frac{n}{n+2}} < 1 \Rightarrow \beta < \alpha^{-2}$