2012 I : QS Transport

1a) $\nu^* = \dfrac{\nu_{eff}}{\omega_b} < 1$ in the banana regime

ω_b is the banana bounce frequency:

$$\tau_b = \frac{2\pi}{\omega_b} = \int \frac{ds}{v_\parallel} \sim \frac{R_0 q}{v_{th}\, \epsilon^{1/2}} \Rightarrow \omega_b \sim \frac{v_{th}\, \epsilon^{1/2}}{R_0 q}$$

ν_{eff} is the effective collision rate:

$$\nu_{eff} \sim \frac{\nu_{ei}^{90^\circ}}{(\Delta\theta)^2} \sim \frac{\nu_{ei}^{90^\circ}}{\epsilon}$$

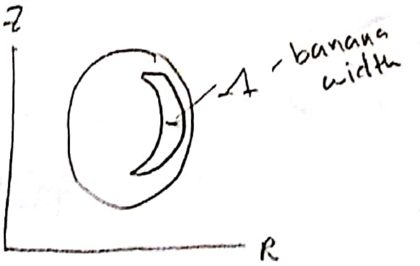
$\nu_{ei}^{90^\circ}$ ← 90° collision rate

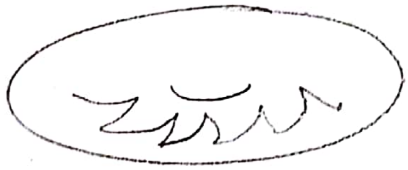
$\epsilon \equiv \frac{r}{R}$

$(\Delta\theta)$: angle of deflection required to untrap

so $\nu^* \sim \dfrac{R_0 q\, \nu_{ei}^{90^\circ}}{v_{th}\, \epsilon^{3/2}} < 1$ in the banana regime.

1b)





3D

2D

2a) In the banana regieme $\sigma_\parallel$ is modified because trapped particles can't carry current. $\sigma_\parallel \Rightarrow \epsilon^{1/2} \sigma_\parallel$ where $\epsilon^{1/2}$ is the trapped fraction.

2b.) $\langle j_s - j_\parallel \rangle \sim j_{BS} - \underbrace{\epsilon^{1/2} \sigma_\parallel E_\parallel}_{\text{neoclassical reduction from (2a)}}$

$$j_{BS} \sim e v_\parallel (n_{c-c}) \sim e v_\parallel \Delta \frac{dn}{dr}$$

n_{c-c}: difference between co- and counter-moving densities

where $\Delta \sim \rho q \epsilon^{-1/2}$, $\rho = \dfrac{v_{th}}{\Omega} = \dfrac{v_{th}}{eB/mc}$, $q = \epsilon \dfrac{B_T}{B_p}$

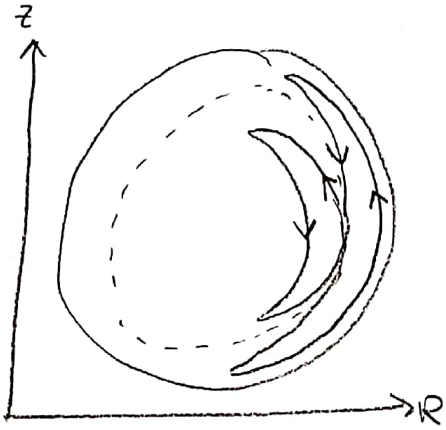
$v_\parallel \sim v_{th}$ because current carried by passing particles.

$$\Rightarrow j_{BS} \sim \cancel{e}\, v_{th} \frac{v_{th}\, mc}{\cancel{e}\cancel{B_T}} \epsilon \frac{\cancel{B_T}}{B_p} \epsilon^{-1/2} \frac{dn}{dr}, \quad v_{th}^2 \sim \frac{T}{m}$$

$$\Rightarrow j_{BS} \sim \epsilon^{1/2} \frac{cT}{B_p} \frac{dn}{dr} \qquad T \frac{dn}{dr} \sim p$$

so $$\boxed{\langle j_{\parallel} - j_s \rangle \sim -\epsilon^{1/2} \left(\frac{cT}{B_p} \frac{dn}{dr} + \sigma_{\parallel} E_{\parallel} \right)}$$

2c) j_{BS} comes from radial gradients and banana orbits:



If $\frac{dn}{dr} < 0$, there are more particles moving on inside banana orbit than the outside orbit, so there is net current along the dotted line. (produced by collisional trapping / detrapping.)

j_{BS} is important b/c (1) helps to confine plasma by supplying additional polodial field and (2) this helps to steepen the pressure profile to make more j_{BS}, which can eventually constitute most of the plasma current.

$\Gamma = -D\nabla n$

3a) $$\Gamma \sim -D_{banana} \left(\frac{dn_0}{dr} - \frac{n_0}{2} \frac{d\ln T_0}{dr} \right) - \Gamma_{wave\text{-}pinch}$$

$$D_{banana} \sim f_T \frac{(\Delta x)^2}{\Delta t} \sim f_T \Lambda^2 \nu_{eff} \sim \epsilon^{1/2} (\rho q \epsilon^{-1/2})^2 \frac{\nu_{ei}^{90°}}{\epsilon} \sim \frac{\rho^2 q^2 \nu_{ei}^{90°}}{\epsilon^{-3/2}}$$

$$\Gamma_W \sim f_T n_0 V_W \sim \epsilon^{1/2} n_0 (V_D \Delta\theta)$$

$F = ma \Rightarrow eE_{\parallel} = m \frac{v_{\parallel}}{\Delta t} \sim m\omega_b v_{\parallel}$

$$\Delta\theta \sim \frac{\Delta v_{\parallel}}{v_{\parallel}} \sim \frac{1}{v_{\parallel}} \left(\frac{eE_{\parallel}}{m\omega_b} \right) \sim \frac{1}{v_{th}\epsilon^{1/2}} \left(\frac{eE_{\parallel}}{m} \right) \left(\frac{R_0 q}{v_{th} \epsilon^{1/2}} \right)$$

$$v_D \sim \frac{1}{\Omega}(\hat{f}\times \mu\nabla B) \sim \frac{\mu}{\Omega}\frac{B_0}{R_0} \sim \frac{1}{\Omega}\frac{v_{th}^2}{\cancel{B}}\frac{\cancel{B}}{R_0} \sim \frac{\rho v_{th}}{R_0}$$

putting everything together yields

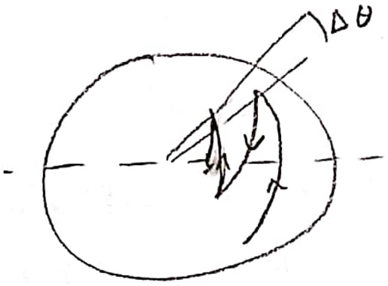
$$\Gamma_w \sim \epsilon^{1/2} n_0 \left(\frac{\rho v_{th}}{\cancel{R_0}}\right)\left(\frac{eE_{\parallel}}{m}\right)\left(\frac{\cancel{R_0}\, q}{v_{th}^2\,\epsilon}\right)$$

$$\sim n_0 \epsilon^{-1/2} \rho q \left(\frac{eE_{\parallel}}{m v_{th}}\right) \sim n_0 \epsilon^{1/2} \frac{B}{B_p}\frac{\cancel{e}E_{\parallel}}{\cancel{m}}\left(\frac{\cancel{m}c}{\cancel{e}B}\right)$$

$$\Gamma_w \sim n_0 \epsilon^{1/2} \frac{cE_{\parallel}}{B_p}$$

so $$\Gamma_{neo} \sim -\frac{\rho^2 q^2 \nu_{ei}}{\epsilon^{-3/2}}\left(\frac{dn_0}{dr} - \frac{n_0}{2}\frac{d\ln T_0}{dr}\right) - n_0 \epsilon^{1/2}\frac{cE_{\parallel}}{B_p}$$

What is the Ware-Pinch?



Because of $E_{\parallel}$, particles spend more time moving // to $E_{\parallel}$, which means they spend more time above the midplane.

$\Rightarrow$ net inward motion!