2013 II : Q1 Transport

(a.) $\nu^* = \frac{\nu_{eff}}{\omega_b} < 1 \qquad \nu_{eff} \sim \nu_{ei}^{90°}/\epsilon \qquad \omega_b \sim \left[\int \frac{ds}{v_\parallel}\right]^{-1} \sim \frac{\epsilon^{1/2} v_{th}}{R_0 q}$

(b.) $\Gamma = -D_{banana}\nabla P - \Gamma_{ware}$

$$D_{banana} = \frac{(\Delta x)^2}{\Delta t} \sim \langle f_T \rangle \Lambda^2 \nu_{eff} \sim \epsilon^{1/2}\Lambda^2 \nu_{ei}^{90°}/\epsilon$$

Find Λ from momentum conservation: $\swarrow \nabla\psi = \nabla R B_p \hat{\phi}$

$$P_\phi = mRv_\phi + eRA_\phi = mRv_\phi - e\psi(r)$$

$$P_t = -e\psi(r) \qquad P_m = m(R_0+r+\Lambda)v_\phi - e\psi(r+\Lambda)$$

Take $\psi(r+\Lambda) \approx \psi(r) + \Lambda \hat{r}\cdot\nabla\psi$

$$= \psi(r) + \Lambda(R_0+r)B_p$$

Then $m(R_0+r+\Lambda)v_\phi - e\cancel{\psi(r)} - e\Lambda(R_0+r)B_p = -\cancel{q\psi(r)}$

$$\Rightarrow \Lambda \sim \frac{m(R_0+\cancel{r})\epsilon^{1/2}v_{th}}{e(R_0+\cancel{r})B_p} \sim q\epsilon^{-1/2}\frac{mv_{th}}{eB_T} \sim \rho q \epsilon^{-1/2}$$

$$\Rightarrow D_{banana} \sim \epsilon^{1/2}(\rho q\epsilon^{-1/2})^2 \nu_{ei}^{90°}/\epsilon \sim \rho^2 q^2 \nu_{ei}^{90°}\epsilon^{-3/2}$$

WHOOPS THEY ASKED FOR CHARGE FLUX LOL...

$$J_{neo} = \langle J_\parallel - J_s \rangle = -J_{BS} - \underbrace{\epsilon^{1/2}\sigma_\parallel E_\parallel}$$

density difference of co- and counter-moving particles

neoclassical correction due to trapped particles not carrying current

passing particles carry current

$$J_{BS} \sim e v_\parallel n_{c-c} \sim e v_{th}\Lambda \nabla n \sim e v_{th}\rho q \epsilon^{-1/2}\frac{dn}{dr}$$

$$\sim v_{th}^2 \frac{mc}{\cancel{B_T}}\epsilon\frac{\cancel{B_T}}{B_p}\epsilon^{-1/2}\frac{dn}{dr} \sim \epsilon^{1/2}\frac{c}{B_p}T\frac{dn}{dr}$$

$$\Rightarrow \boxed{J_{neo} \sim -\epsilon^{1/2}\left[\frac{c}{B_p}T\frac{dn}{dr} - \sigma_\parallel E_\parallel\right]}$$

(c.) The formal derivation of the bootstrap we use the Drift kinetic equation, then expand $f = f_{11} + f_1$, where $f_1 = f_s + \hat{f}$. Use this to calculate J_s and J_{BS}.

Drif kinetic equation: $(\hat{n} v_{11} + \vec{v}_D) \cdot \nabla f + \frac{e}{m} E_{11} V_{11} \frac{\partial f}{\partial E} = C_L[f]$

(d.) The difference in density of the co- and counter-moving trapped particles dictates the bootstrap current by collisional traping and detrapping of the passing particles that actually carry the current.

(e.) Bootstrap current is important because

(1) creates additional B_p → increased confinment

(2) creates additional J_T for "free" ie no external heating.

PART II: Drift waves

(a.) $$1 - \frac{\omega_{*e}}{\omega} + b_s - \frac{k_{11}^2 c_s^2}{2\omega^2}\left(1 - \frac{\omega_{*i}}{\omega}\left[\overbrace{\cancel{1}}^{O:(1 \ll \eta_i)} + \eta_i\right]\right) = 0$$

$$\omega_* = k_y V_* \sim k_y \left(\frac{cT}{eB_0}\right)\frac{d\ln(n_0)}{dx}$$

memorize these.

Take $\overset{\text{Fluid limit}}{V_{thi}} \ll \frac{\omega}{k_{11}} \ll \overset{\text{adiabatic limit}}{V_{the}}$ (If η_i) big enough nothing else matters?)

$$\Rightarrow 1 + \frac{k_{11}^2 c_s^2}{2\omega^3}\omega_{*i}\eta_i = 0 \Rightarrow \omega = \left(-\frac{2}{k_{11}^2 c_s^2 \omega_{*i}\eta_i}\right)^{1/3}$$

ω has to complex solutions, are with $Im(\omega) > 0$
→ there is an unstable mode!

(b.) Quasineutrality is justified because the electrons are adiabatic and so they act as a neutralizing agent.