2013 II: Q4B Math

$$\frac{d^2\psi}{dx^2} + Q(x)\psi = 0$$

a.) Let $Q(x)$ be real and finite. The we can write

$$(\psi^*)'' + Q(x)\psi^* = 0$$

Then $\left(\psi^*\psi'' + \cancel{\psi^* Q(x)\psi}\right) - \left(\psi(\psi^*)'' + \cancel{\psi Q(x)\psi^*}\right) = 0$

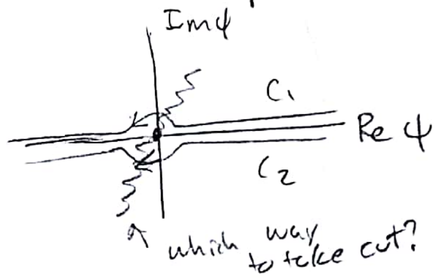
$$\Rightarrow \frac{d}{dx}\left(\psi^*\psi' - (\psi^*)'\psi\right) = 0 \qquad \cancel{\psi^{*\prime}\psi'} + \psi^*\psi'' - \psi^{*\prime\prime}\psi - \cancel{\psi^{*\prime}\psi'}$$

Which is a flux conservation law equivalent to

$$\underline{\mathrm{Im}(\psi^*\psi') = \text{const.}}$$

b.) Let $Q(x) = \frac{a}{x}$ with $a > 0$ $\Rightarrow$ absorbtion problem

Near the pole $\psi'' = -\frac{a}{x}\psi$ $\qquad y^* \sim -iy_0$



$$\int_{C_1} \psi'' dz = [\psi']_-^+ = \pi i a y_0$$

$$\int_{C_2} \psi'' dz = [\psi']_-^+ = -\pi i a y_0$$

Then $\mathrm{Im}(y^* y') \sim \pi a |y_0|^2$ if cut taken below pole

$\mathrm{Im}(y^* y') \sim -\pi a |y_0|^2$ if cut taken above pole

$\Rightarrow$ pole can be a source or a sink of flux if the brach cut is placed above or below the pole. Causality requires this be a sink, so need to place cut above the pole $\Rightarrow$ same as Landau prescription for Landau Damping.