2013 I : 1 Waves

$$\chi_s = \frac{2\omega_p^2}{k^2 v_{th}^2}\left[1 + \frac{\omega - kV_D}{kv_{th}} Z(\xi)\right]$$

drift velocity $V_D = 0$, $\quad \xi = \dfrac{\omega}{kv_{th}}$

$$\varepsilon = 0 = 1 + \sum_s \chi_s = 1 + \frac{2\omega_{pe}^2}{k^2 v_{the}^2}\Big[1 + \underbrace{\frac{\omega}{kv_{the}} Z(\xi_e)}_{\text{expand w/ } \xi_e \ll 1}\Big] + \frac{2\omega_{pi}^2}{k^2 v_{thi}^2}\Big[\underbrace{1 + \frac{\omega}{kv_{thi}} Z(\xi_i)}_{\text{expand w/ } \xi_i \gg 1}\Big]$$

Then $\chi_e \approx \dfrac{2\omega_{pe}^2}{k^2 v_{the}^2}\left[1 + \cancel{\dfrac{\omega}{kv_{the}}\left(-2\dfrac{\omega}{kv_{the}}\right)}\right] \quad (\xi_I \ll \xi_R)$

$$\approx \frac{2}{k^2 \lambda_{De}^2} \qquad \left(\frac{\omega}{kv_{the}}\right)^2 \ll 1$$

$$\chi_i \approx \frac{2\omega_{pi}^2}{k^2 v_{thi}^2}\left[1 + \frac{\cancel{\omega}}{\cancel{kv_{thi}}}\left(-\frac{\cancel{kv_{thi}}}{\cancel{\omega}}\right)\left(1 + \frac{1}{2}\left(\frac{kv_{thi}}{\omega}\right)^2\right)\right]$$

$$\approx \frac{2\omega_{pi}^2}{k^2 v_{thi}^2}\left[1 - 1 - \frac{1}{2}\left(\frac{kv_{thi}}{\omega}\right)^2\right] \approx -\frac{\omega_{pi}^2}{\omega^2}$$

$$\Rightarrow \varepsilon = 1 - \frac{\omega_{pi}^2}{\omega^2} + \frac{2}{k^2\lambda_{De}^2} = 0$$

$$\omega^2 = \omega_{pi}^2\left(\frac{1/2}{\frac{1}{2} + \frac{1}{k^2\lambda_{De}^2}}\right) = \frac{1}{2}\,\frac{\omega_{pi}^2 \lambda_{De}^2 k^2}{\frac{1}{2}(k^2\lambda_{De}^2 + 1)}$$

take $\lambda_{De}^2 \omega_{pi}^2 = C_s^2$ to get

$$\omega^2 = C_s^2 k^2 \left(\frac{1}{1 + 2k^2\lambda_{De}^2}\right)$$

NOTE: for IAW, also take $k^2\lambda_{De}^2 \ll 1$

$$\Rightarrow \boxed{\omega^2 = C_s^2 k^2}$$