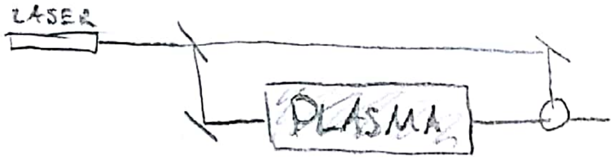
2013 I : Q2 Diagnostics

1.)



A typical interferometer works by comparing the phase shift of a signal that propagated through a plasma to the same signal just through vacuum. With $\Phi = K\lambda \int N_e dl$, you can extract info about the line-averaged density. Limitations: cannot be used if there exists any $n(x) > n_c$ in the plasma, where n_c is the cutoff density. Cannot determine density profiles. Sensitive to vibrations (see part 2.)

2.) $\lambda_c = 10.6 \mu m$, $\lambda_{He} = 0.633 \mu m$. $\omega >> \omega_p$.

(a.) phase shift is given by $\Phi_{meas} = \Phi_{plasma} + \Phi_{vib}$

$\Phi_{vib} = 2\pi \frac{L}{\lambda}$ where L is vibrated distance (same for both)

so $2\pi L = 2\pi L \Rightarrow \lambda_c \Phi_c - K\lambda_c^2 \int N_e dl = \lambda_{He}\Phi_{He} - K\lambda_{He}^2 \int N_e dl$

$$\Rightarrow \boxed{\int N_e dl = \frac{\lambda_c \Phi_c - \lambda_{He}\Phi_{He}}{K(\lambda_c^2 - \lambda_{He}^2)}}$$

(b.) since $\Phi_{vib} = 2\pi \frac{L}{\lambda}$, assume $\Phi_c << \Phi_{He}$ Let $n = \int N_e dl$

$$\delta n \approx \frac{\lambda_{He}\, \delta\Phi_{He}}{K(\lambda_c^2 - \lambda_{He}^2)} \Rightarrow \boxed{\delta n \approx \frac{\pi}{K} \frac{\lambda_{He}}{\lambda_c^2 - \lambda_{He}^2}}$$

(c.) $\frac{\delta n}{n} = \frac{\pi}{k} \frac{\lambda_{He}}{\lambda_c^2 - \lambda_{He}^2} \underbrace{\frac{k(\lambda_c^2 - \lambda_{He}^2)}{\lambda_c \Phi_c - \lambda_{He}\Phi_{He}}}$

not given Φ_c, Φ_{He}. use n

$$\frac{\delta n}{n} = \frac{\pi}{k} \frac{\lambda_{He}}{\lambda_c^2 - \lambda_{He}^2} \frac{1}{(n = 10^{14}\,cm^{-3})(100\,cm)}$$ need k.

The phase shift is given by (with $k = \frac{2\pi}{\lambda}$, $N = \frac{ck}{\omega}$)

$$\Phi = \int |(k_{plasma} - k_0)| d\ell = \frac{\omega}{c}\int |(N-1)| d\ell$$

$\sqrt{1-x} \approx 1 + x\frac{1}{2!}$

Use $\omega^2 = \omega_p^2 + c^2k^2 \Rightarrow N^2 = 1 - \frac{\omega_p^2}{\omega^2}$ to get

$$\Phi = \left|\frac{\omega}{c}\int d\ell \sqrt{1 - \frac{\omega_p^2}{\omega^2}} - 1\right| \approx \frac{\omega}{c}\int \frac{\omega_p^2}{2\omega^2} d\ell$$

Note that $\omega = \omega_p$ occurs at n_c : $\omega^2 = \frac{4\pi n_c e^2}{m_e}$

$$\Rightarrow n_c = \frac{m_e \omega^2}{4\pi e^2} \Rightarrow \frac{\omega_p^2}{\omega^2} = \frac{N_e}{n_c}$$

$\frac{\omega}{k} = c$

$$\Rightarrow \Phi = \frac{\omega}{2cn_c}\int N_e\, d\ell = \frac{\cancel{4}\pi e^2}{\cancel{2} m_e c \cancel{\omega}} \frac{\lambda}{\cancel{\lambda}} \int N_e\, d\ell$$

$\frac{2\pi}{\omega\lambda} = \frac{1}{c}$

$$\Rightarrow k = \frac{e^2}{m_e c^2} \approx \frac{25\times10^{-20}}{9\times10^{-28}\times9\times10^{20}} \approx 10^{-12}\ cm^2$$

$$\frac{\lambda_{He}}{\lambda_c^2 - \lambda_{He}^2} \approx \frac{1}{100}\ \frac{1}{\mu m} = 10^{-2}\ \frac{10^6\,\mu m}{m}\ \frac{m}{10^2\,cm} = 10^2\ cm^{-1}$$

$\frac{\pi}{k}$ $\frac{\lambda}{\lambda^2 - \lambda^2}$ $\frac{1}{n_e}$ $\frac{1}{L}$

$$\Rightarrow \frac{\delta n}{n} \sim (10^{12}\ cm^{-2})(10^2\ cm^{-1})(10^{-14}\ cm^3)(10^{-2}\ cm^{-1})$$

$$\frac{\delta n}{n} \sim 10^{-2} \Rightarrow \boxed{\frac{\delta n}{n} \sim 1\%}$$