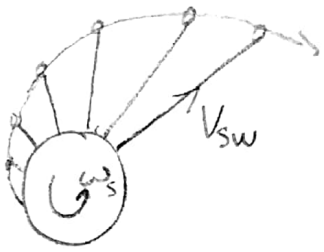
2014 II: Q2 MHD



Field lines frozen in to stream lines

$$\Rightarrow \frac{B_\phi}{B_r} = \frac{u_\phi}{u_r} = \frac{r\,d\phi}{dr}$$

$$u_\phi = \omega_s r \Rightarrow \frac{d\phi}{dr} = \frac{\omega_s}{v_{sw}} \Rightarrow \boxed{\phi(r) = \left(\frac{\omega_s}{v_r}\right) r + \phi_0}$$

$$\boxed{\frac{B_r}{B_\phi} = \frac{v_{sw}}{\omega_s r}}$$

Prove solar wind exists:

Assume $u = 0$. Then $\rho \frac{Du}{Dt} = -\frac{dp}{dr} - \rho \frac{GM}{r^2} = 0$ ($\frac{Du}{Dt} \to 0$)

$$\Rightarrow \frac{dp}{dr} = -\rho \frac{GM}{r^2} \quad \text{but} \quad \rho = \frac{m}{2T} p \Rightarrow \frac{1}{p}\frac{dp}{dr} = -\frac{GMm}{2T}\frac{1}{r^2}$$

$$\Rightarrow p = C \exp\left(\frac{GMm}{2T}\frac{1}{r}\right)$$

At solar surface: $p = p_0 \Rightarrow p(r) = p_0 \exp\left[\frac{GMm}{2T}\left(\frac{1}{r} - \frac{1}{R_s}\right)\right]$

At $r >> R_s$, this becomes $p(r) \sim p_0 \exp\left(-\frac{GMm}{2TR_s}\right) > 0$

But we know that $p(r) \approx 0$ at ISM! so there is no static equilibria $\longrightarrow$ so solar wind exists.