2014 II : Q4 GPP

(a.) Assume each atom has one free electron

Then $n_{Au} = \frac{\rho}{M} = \frac{19.3 \text{ g/cc}}{197 \cdot 1.7 \times 10^{-24} \text{ g}} \sim 10^{23} \text{ cm}^{-3}$

Consider the electron sea as plasma. $\omega^2 = \omega_p^2 + c^2 k^2$

$$\omega_{pe} = \left(\frac{4\pi n_{Au} e^2}{m_e}\right)^{1/2} \sim 5.6 \times 10^4 \sqrt{10^{23}} \frac{\text{rad}}{\text{sec}} \qquad 23/2 = 11.5$$

$$\sim 5.6 \times 10^{15.5} \frac{\text{rad}}{\text{sec}}$$

If $\omega^2 - \omega_p^2 < 0$, k is imaginary $\Rightarrow$ wave reflected.

$$\omega < 5.6 \times 10^{15.5} \frac{\text{rad}}{\text{sec}} \Rightarrow f < 10^{15} \text{ Hz}$$

$$\lambda = \frac{c}{f} \Rightarrow \lambda > \frac{3 \times 10^{10} \text{ cm/s}}{10^{15} \text{ /s}} \sim 3 \times 10^{-5} \text{ cm} \sim 300 \text{ nm}$$

300nm is UV range, so all visible light is reflected.
(360–700 nm)

(b.) $k = \frac{1}{c}\sqrt{\omega^2 - \omega_p^2} = \frac{i}{c}\sqrt{\omega_p^2 - \omega^2} \quad (\omega_p^2 > \omega^2)$

The wave goes like $\sim \exp[i(kx - \omega t)]$ so the amplitude goes like $\sim \exp\left(-\frac{x}{c}\sqrt{\omega_p^2 - \omega^2}\right) \sim \exp\left(-\frac{x}{L}\right)$ where L is a characteristic e-folding length scale $L = c/\sqrt{\omega_p^2 - \omega^2}$, $\omega_p^2 > \omega^2$. If the foil is thinner than this, some light will get through.

(c.) For blue light, $\lambda \sim 400\text{nm} \Rightarrow f = \frac{c}{\lambda} \sim \frac{3 \times 10^{10}}{4 \times 10^{-5}} \sim \frac{3}{4} \times 10^{15} \text{ Hz}$

$$\Rightarrow L_B = \frac{3 \times 10^{10} \text{ cm/s}}{\sqrt{36 \times 10^{15} \underbrace{(1 - 3/4)}_{1/4}}} \sim \sqrt{\frac{4}{1}}\, c'$$

For red light, $\lambda \sim 700\text{nm} \Rightarrow f = \frac{c}{\lambda} \sim \frac{3}{7} \times 10^{15} \text{ Hz}$

$$\Rightarrow L_R \sim \frac{1}{\sqrt{1 - 3/7}}\, c' \sim \sqrt{\frac{7}{4}}\, c'$$

so $L_B > L_R$

So the light that gets through is *blue-ish*!