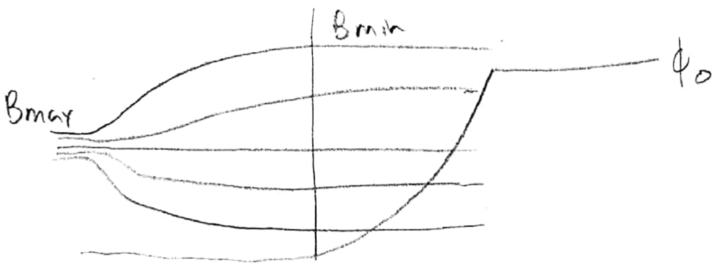
2014 I : Q1 GPP

$$B = B_0 \begin{cases} 1 + (R-1)\left(\frac{z^2}{L^2}\right) & -L < z < 0 \\ 1 & z \geq 0 \end{cases}$$

$$\phi = \begin{cases} 0 & z < 0 \\ \phi_0 \left(\frac{z^2}{L^2}\right) & 0 \leq z < L \\ \phi_0 & z > L \end{cases}$$



a.) Ions and electrons are trapped by μ invariance and energy conservation for $-L < z < 0$. Ions are repeled by the potential $\phi_0 > 0$ for $z > L$, and are thus trapped. Electrons, however, are accelerated out of the machine for $z > L$.

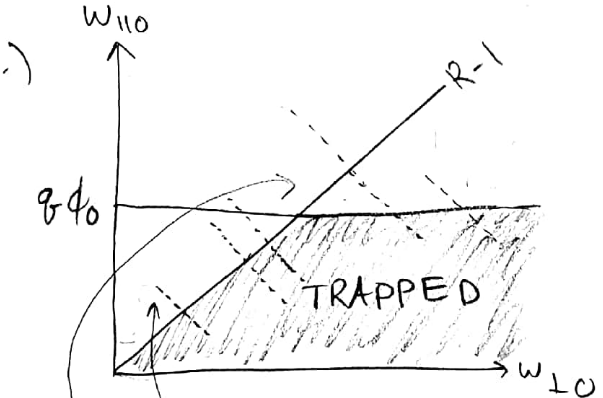
b.) For $-L < z < 0$, trapped if $\mu B_{max} > \mu B_{min} + W_{\parallel 0}$

$$\Rightarrow W_{\perp 0}(R-1) > W_{\parallel 0} \Rightarrow \underline{\frac{W_{\parallel 0}}{W_{\perp 0}} < R-1}$$

For $0 \leq z < L$, trapped if $\underline{q\phi_0 > W_{\parallel 0}}$

NEED BOTH SIMULTANEOUSLY

c.)



Pitch angle scattering occurs along dotted lines (constant energy). The side of the device the ions leave on depends on (1) the ion energy and (2) the size off the scatter.

leaves on left if $W_{\perp 0} + W_{\parallel 0} < q\phi_0$

can leave on either side depending on size of scatter d.) $\phi = \phi(t)$, $B = B(t)$ (slowly varrying $\Rightarrow$ conserve adiabatic invariants.

$$J = \int_{-z_m}^{z_e} v_\parallel(s)\, ds$$

For $z<0$, $W_\parallel + W_\perp = W_{\perp 0} + W_{\parallel 0}$

$$v_\parallel^2 = \frac{2}{m}\left[W_{\perp 0}\left(1-\frac{B}{B_0}\right) + W_{\parallel 0}\right]$$

$$v_\parallel(s) = \sqrt{\frac{2}{m}}\left[W_{\perp 0}(R-1)\frac{z^2}{L^2} + W_{\parallel 0}\right]^{1/2}$$

$$\int_0^{z_m} dz \sqrt{\frac{2}{m}}\left[W_{\perp 0}(R-1)\frac{z^2}{L^2} + W_{\parallel 0}\right]^{1/2}$$

$$z_m^2 = L^2\left(\frac{W_{\parallel 0}}{W_{\perp 0}}\right)\frac{1}{1-R}$$

$$= \int_0^{z_m} dz \sqrt{\frac{2}{m}}\, W_{\parallel 0}^{1/2}\left[1-\left(\frac{z}{z_m}\right)^2\right]^{1/2} = \sqrt{\frac{2}{m}}\frac{\pi}{4} W_{\parallel 0}^{1/2} z_m$$

Similarity, for $z>0$, $W_\parallel = W_{\parallel 0} - q\phi_0 \frac{z^2}{L^2}$ $\qquad z_E^2 = W_{\parallel 0} L^2 / q\phi_0$

$$\int_0^{z_E} dz \sqrt{\frac{2}{m}}\left(W_{\parallel 0}^{1/2}\left(1-\frac{z^2}{z_E^2}\right)^{1/2}\right) = \sqrt{\frac{2}{m}}\frac{\pi}{4} W_{\parallel 0}^{1/2} z_E$$

$$\Rightarrow \boxed{J = W_{\parallel 0}^{1/2}(z_m + z_E) = \text{const.}}$$

$R = \frac{B_{max}}{B_0}$

e.) $\phi_0(t=T) = \gamma\phi_0(t=0)$ and $B_0(t=T) = \alpha B_0(t=0)$

$$\mu = \frac{W_{\perp 0}}{B_0} = \frac{W_{\perp 0}'}{\alpha B_0} \Rightarrow W_{\perp 0}' = \alpha W_{\perp 0}$$

$$J = W_{\parallel 0} L\left(\frac{1}{\sqrt{W_{\perp 0}(R-1)}} + \frac{1}{\sqrt{q\phi_0}}\right) \Rightarrow W_{\parallel 0}\left(\frac{1}{\sqrt{W_{\perp 0}}} + \sqrt{\frac{R-1}{q\phi_0}}\right) = \text{const.}$$

$$\Rightarrow W_{\parallel 0}' = W_{\parallel 0}\underbrace{\left(\frac{1}{W_{\perp 0}^{1/2}} + \sqrt{\frac{R-1}{q\phi_0}}\right)\left(\frac{1}{\sqrt{\alpha}W_{\perp 0}^{1/2}} + \sqrt{\frac{R-1}{q\gamma\phi_0}}\right)^{-1}}_{\equiv \beta} = \beta W_{\parallel 0}$$

We want untrapped on $z>0$, so require

$$W_{\parallel 0}' \geq q\gamma\phi_0 \quad \text{and} \quad W_{\parallel 0}' < W_{\perp 0}'\sqrt{R-1}$$

$$\Rightarrow q\gamma\phi_0 \leq \beta W_{\parallel 0} < \alpha W_{\perp 0}\sqrt{R-1}$$

initially trapped: $q\phi_0 > W_{\parallel 0}$

i) To become untrapped for $z < 0$ (with $\omega_{\perp 0} \gg \omega_{\parallel 0}$ and $\omega_{\perp 0} \gg q\phi_0$) we require $\omega'_{\parallel 0} < q\gamma\phi_0$, $\omega'_{\parallel 0} > \omega'_{\perp 0}\sqrt{R-1}$.

$$\Rightarrow \alpha\omega_{\perp 0}\sqrt{R-1} < \beta\omega_{\parallel 0} < q\gamma\phi_0$$

How to achieve this? one way is just to let $\alpha \to 0$.
(ie slowly turn off the magnetic field.)