2015 II : Q1 Waves

a.) The absorbed power is $P_{abs} = \frac{\omega}{8\pi} \mathcal{E}^* \,\epsilon_A(b, \vec{x}, \omega, \vec{k})\, \mathcal{E}$

absorbed power density

parallel propagation: $N^2 = R$, $N^2 = L$

$$R = 1 - \sum_s \frac{\omega_{ps}^2}{\omega(\omega + \Omega_s)} \qquad L = 1 - \sum_s \frac{\omega_{ps}^2}{\omega(\omega - \Omega_s)}$$

cyclotron heating of minority ions occurs with L-wave

(cold plasma)

$$L = \cancel{1} - \cancel{\frac{\omega_{pe}^2}{\omega(\omega+\Omega_e)}} - \cancel{\frac{\omega_{pi}^2}{\omega(\omega-\Omega_i)}} - \frac{\omega_{pm}^2}{\omega(\omega-\Omega_m)} \qquad \Omega_m = \underset{\underset{\omega}{\parallel}}{\frac{eB_0}{mc}} \left(1 + z/R_0\right)$$

<< resonance

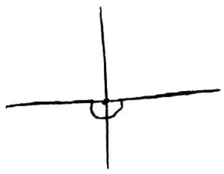
$$\mathcal{E} = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix} \Rightarrow \mathcal{E}_A = \begin{pmatrix} \mathrm{Im}\,S & -i\,\mathrm{Im}\,D & 0 \\ i\,\mathrm{Im}\,D & \mathrm{Im}\,S & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{aligned} D &= \tfrac{1}{2}(R-L) \\ S &= \tfrac{1}{2}(R+L) \end{aligned}$$

$$\Rightarrow \mathcal{E}_A = \frac{1}{2} \begin{pmatrix} 1 & i & 0 \\ -i & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathrm{Im}\,L$$

$$\mathrm{Im}\,L = \mathrm{Im}\left[\frac{-\omega_{pm}^2}{\omega(\omega - \omega(1 + z/R_0))}\right] = \frac{-\omega_{pm}^2}{\omega^2} \mathrm{Im}\left[\frac{1}{-z/R_0}\right] = \frac{\omega_{pm}^2 R_0}{\omega^2} \mathrm{Im}\left[\frac{1}{z}\right]$$

$E_x + iE_y \quad -iE_x + E_y$

$$\text{so } P_{abs} = \frac{\omega}{8\pi} \frac{\omega_{pm}^2 R_0}{2\omega^2} \begin{pmatrix} E_x^* & E_y^* \end{pmatrix} \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} \mathrm{Im}\left[\frac{1}{z}\right]$$



$$P = \int P_{abs}\, dz = \frac{\omega_{pm}^2 R_0}{16\pi\omega} \underbrace{\left(|E_x|^2 + iE_y E_x^* - iE_x E_y^* + |E_y|^2\right)}_{= |\xi_+|^2} \underbrace{\mathrm{Im}\left[\int \frac{1}{z}\, dz\right]}_{= \mathrm{Im}[i\pi]}$$

$$\Rightarrow \boxed{P = \left(\frac{\omega_{pm}^2 R_0}{16\omega} |\xi_+|^2\right)\Big|_{z=0}}$$

for the residue

b.) To guess the absorbtion region a_{abs} try

$$P \sim \exp\left[-\left(\frac{\omega - \Omega_m}{k_{\parallel} v_{thm}}\right)^2\right] = \exp\left[-\left(\frac{\omega z}{k_{\parallel} v_{th} l_0}\right)^2\right]$$

so $\boxed{a_{abs} \approx \frac{k_{\parallel} v_{th} l_0}{\omega}}$

c.) I'm super smart and obviously know the relevant parameters.

$$\left.\begin{aligned} k_{\parallel} l_0 &= 2\pi \\ \omega &\sim 2\pi \times 50 \text{ MHz} \\ T &\sim 10^4 \text{ eV} \\ m_m &\sim 3 m_p \end{aligned}\right\} \Rightarrow \boxed{a_{abs} \sim 1 \text{ cm}}$$