2015 I : QS Kinetic

$$\frac{\partial f}{\partial t} + u \cdot \nabla_x f + \frac{q\vec{E}}{m} \cdot \nabla_v f = 0 \qquad \vec{E} = -\nabla\phi \quad \nabla^2\phi = -4\pi e(n_i - n_e)$$

$$\frac{\partial}{\partial t}\Big[\underbrace{\int \tfrac{1}{2}mv^2 f\, d^3v}_{K}\Big] = -\int d^3v\, \tfrac{1}{2}mv^2 \Big[\underbrace{\vec{v}\cdot\nabla f}_{\nabla\cdot\vec{Q}} - \frac{\overset{-e}{\cancel{q}}}{m}\nabla\phi \cdot \frac{\partial f}{\partial \vec{v}}\Big]$$

$$\frac{dK}{dt} = -\nabla\cdot\Big(\underbrace{\int d^3v\, \tfrac{1}{2}mv^2\vec{v} f}_{\vec{Q}}\Big) - \frac{e}{m}\int \tfrac{1}{2}mv^2\, \nabla\phi\, \frac{\partial f}{\partial \vec{v}}\, d\vec{v}$$

IBP

$$= -\nabla\cdot\vec{Q} - \cancel{\Big[\frac{e}{m}\nabla\phi\,\tfrac{1}{2}mv^2 f\Big]_{-\infty}^{\infty}}^{\,0} + \frac{e}{\cancel{m}}\int d^3v\, \nabla\phi \cdot \cancel{m}\vec{v}\, f(v)$$

$$\frac{dK}{dt} = -\nabla\cdot\vec{Q} + e\nabla\phi\cdot(n\vec{u}) = -\nabla\cdot\vec{Q} + \vec{E}\cdot\vec{J}$$

$$U = \frac{|E|^2}{8\pi} = \frac{|\nabla\phi|^2}{8\pi} \Rightarrow \frac{dU}{dt} = \frac{d}{dt}\frac{|E|^2}{8\pi}$$

0 by divergence theorem.

$$\frac{d\,\text{Energy}}{dt} = \int d^3x\left(\frac{dK}{dt} + \frac{du}{dt}\right) = \int d^3x\left(-\cancel{\nabla\cdot\vec{Q}} + \vec{E}\cdot\vec{J} + \frac{d}{dt}\frac{|E|^2}{8\pi}\right)$$

0 by divergence theorem

$$= \int d^3x\, \vec{E}\cdot\vec{J} + \frac{1}{8\pi}\frac{d}{dt}\left(\cancel{\nabla\cdot(\phi\nabla\phi)} - \phi\nabla^2\phi\right)$$

$$= \int d^3x\, \vec{E}\cdot\vec{J} + \frac{1}{\underset{2}{\cancel{8\pi}}}\frac{d}{dt}\phi\left(\cancel{4\pi}e(n_0 - n_e)\right) \qquad \int d^3v\, f$$

$$= \int d^3x\, \vec{E}\cdot\vec{J} + \frac{e}{2}\frac{d\phi}{dt}(n_0 - n_e) + \frac{e}{2}\frac{dn_e}{dt}$$

but $\frac{dn_e}{dt} = -\nabla\cdot(n\vec{u})$ from continuity equation. 0 by divergence theorem!

$$= \int d^3x\, \vec{E}\cdot\vec{J} + \frac{e}{2}\left[\frac{d\phi}{dt}(n_0 - n_e) - \cancel{\nabla\cdot(n\vec{u})}\right]$$

Hmmm.

back to

$$\frac{dE}{dt} = \int d^3x \ \vec{E} \cdot \vec{J} + \frac{du}{dt}$$

note that Poynting's Theorem is

$$\frac{du}{dt} + \underbrace{\int_S \vec{N} \cdot d\vec{S}}$$
$$= -\int_V dv \ \vec{J} \cdot \vec{E}$$

$$\vec{N} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = 0$$

so $\frac{du}{dt} = -\int_V dv \ \vec{J} \cdot \vec{E}$

$\Rightarrow \frac{d\,\text{Energy}}{dt} = 0 \Rightarrow$ Energy is conserved!