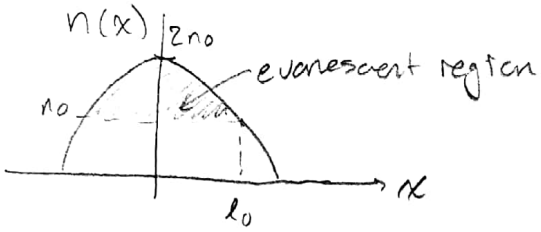
2015 I: Q7 Waves

$n(x) = \left(2 - \frac{x^2}{\ell_0^2}\right) n_0$ O-wave $\Rightarrow \omega^2 = \omega_p^2 + c^2k^2 \Rightarrow k = \frac{1}{c}\sqrt{\omega^2 - \omega_p^2}$

Assume $E \sim e^{ikx} = e^{i\frac{x}{L}}$ where $L = c/\sqrt{\omega^2 - \omega_p^2}$

$\omega_p^2 = \frac{4\pi e^2}{m} n = \frac{4\pi e^2 n_0}{m}\left(2 - \frac{x^2}{\ell_0^2}\right)$ n_0 is critical density



↳ occurs when $\omega^2 = \omega_p^2$

$\Rightarrow \omega^2 = \frac{4\pi e^2 n_0}{m}$

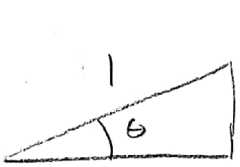
$\Rightarrow \frac{\omega_p^2}{\omega^2} = \left(2 - \frac{x^2}{\ell_0}\right)$

So $L = \frac{c/\omega}{\sqrt{1 - \omega_p^2/\omega^2}} = \frac{4\pi e^2 n_0 c}{m}\left(\frac{x^2}{\ell_0^2} - 1\right)^{-1/2}$

$\Rightarrow E(x) \sim e^{i\frac{\omega}{c}\left(\frac{x^2}{\ell_0^2} - 1\right)^{1/2} x}$?

Try WKB? $E(x) = E_0 \exp\left(\int_0^x ik(x')\,dx'\right)$

$= E_0 \exp\left(\int_{-\ell_0}^{\ell_0} i\frac{\omega}{c}\left(\frac{x^2}{\ell_0^2} - 1\right)^{1/2} dx\right) = E_0 \exp\left(\frac{\omega}{c}\int_{-\ell_0}^{\ell_0}\left(1 - \frac{x^2}{\ell_0^2}\right)^{1/2} dx\right)$



$\sin\theta = \frac{x}{\ell_0}$ $dx = \ell_0 \cos\theta\, d\theta$

$x = \pm\ell_0 \Rightarrow \theta = \pm\frac{\pi}{2}$

$\Rightarrow E(x) = E_0 \exp\left(\frac{\omega}{c}\int \ell_0 \cos^2\theta\, d\theta\right)$

$\frac{E(x)}{E_0} = \exp\left(\frac{\ell_0\omega}{c}\left[\frac{\theta}{2} + \frac{1}{4}\sin(2\theta)\right]\right)$

$\Rightarrow \frac{E(x)}{E_0} = \exp\left(\frac{\pi}{2}\frac{\omega\ell_0}{c}\right)$

$\Rightarrow \frac{|E_x|^2}{|E_0|^2} = T = \exp\left(\frac{\pi\omega\ell_0}{c}\right)$

$\Rightarrow \boxed{\ln(T) = \frac{\pi\omega\ell_0}{c}}$