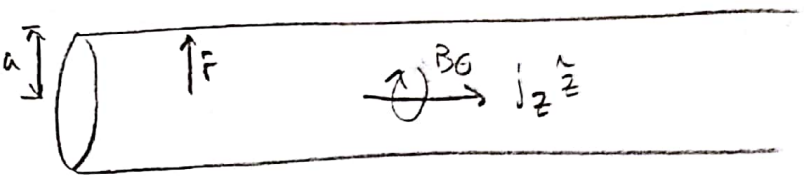
2016 II: Q3 GPP (MHD)

1a.)



$\vec{J} \times \vec{B} = \nabla P \qquad \nabla \cdot \vec{B} = 0 \qquad J = \frac{1}{\mu_0} \nabla \times \vec{B} \qquad \vec{B} = B(r)\hat{\theta}$

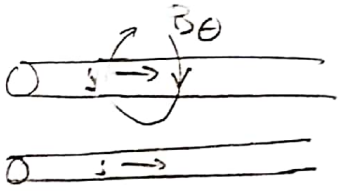
$$B_\theta = \frac{\mu_0 \pi r^2 j_z}{2\pi r} = \frac{\mu_0 j_z}{2} r$$

$$\frac{d}{dr} P = -\frac{\mu_0 j_z^2}{2} r \Rightarrow P = \frac{\mu_0 j_z^2}{2}\left(-\frac{r^2}{2}\right) - P_0$$

$$P(r=a) = 0 \Rightarrow P_0 = \frac{\mu_0 j_z^2}{4} a^2$$

$$\Rightarrow \boxed{P(r) = \frac{\mu_0 j_z^2}{4}(a^2 - r^2)}$$

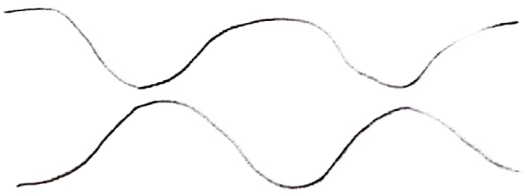
b.)



$\vec{J} \times \vec{B} = JB \uparrow$

So the two are attracted

c.) Z-pinch is susceptible to sausage instability



and to the kink instability