2016 II : Q6 Neoclassical

a.) $P_\zeta = mRv_\zeta + \frac{e}{c}\psi \approx mRv_{\parallel} + \frac{e}{c}\psi(r,t) = \text{const.}$

$$\frac{d}{dt}P_\zeta = 0 = \frac{d}{dt}(mRv_{\parallel}) + \underbrace{\frac{e}{c}\frac{d}{dt}\psi(r,t)}_{\frac{e}{c}\left(\frac{\partial}{\partial t}\psi(r,t) + \frac{\partial r}{\partial t}\frac{\partial \psi}{\partial r}\right)}$$

$$\left\langle -\cancel{m\frac{d}{dt}(Rv_{\parallel})} \right\rangle^{0} = \frac{e}{c}\left\langle \frac{\partial \psi}{\partial t} + \vec{v}\cdot\nabla\psi \right\rangle$$

$$\Rightarrow \left\langle \frac{\partial \psi}{\partial t} \right\rangle = -\langle \vec{v}\cdot\nabla\psi \rangle$$

b.) $\nabla \times E = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \Rightarrow \oint E\cdot dl = -\frac{1}{c}\frac{\partial}{\partial t}\int_S \vec{B}\cdot d\vec{s}$

$\vec{B} = \nabla \times A \Rightarrow \oint \vec{E}\cdot \partial\vec{l} = -\frac{1}{c}\frac{\partial}{\partial t}\oint \vec{A}\cdot d\vec{l}$

$$E_\zeta = -\frac{1}{c}\frac{\partial A_\zeta}{\partial t}$$

But $\nabla\psi = RB_p \Rightarrow \psi = -RA_\zeta \Rightarrow E_\zeta = +\frac{1}{cR}\frac{d\psi}{dt}$

$$\langle E_\zeta \rangle = \frac{1}{cR}\left\langle \frac{d\psi}{dt}\right\rangle = -\frac{1}{cR}\langle \vec{v}\cdot\nabla\psi\rangle = -\frac{1}{c\cancel{R}}\langle \vec{v}\cdot \cancel{R}\vec{B}_p\rangle$$

$$\Rightarrow \boxed{v = -\frac{cE_\zeta}{B_p}}$$

c.) $\nabla\times\vec{A} = \vec{B}$, $\psi = -RA_\zeta \Rightarrow P_\zeta = mRv_\zeta - \frac{eR}{c}A_\zeta$

ψ is flux. $\psi = \int \vec{B}\cdot d\vec{s}$

$$P_\zeta = mRv_\zeta - \frac{e}{c}R\int \vec{B}\cdot d\vec{s}$$

d.) $P_\zeta = m(r + R_0 + \Lambda) v_\zeta + \frac{e}{c}\psi(r) + \frac{e}{c}\Lambda \psi'(r)$

$$P_\zeta = m(r + R_0 + \Lambda) v_\zeta + \frac{e}{c}\psi(r) + \frac{e}{c}\Lambda (R_0 + r) B_p$$

e.) At banana location $P_\zeta = \frac{e}{c}\psi(r)$

$$\Rightarrow m(r + R_0 + \Lambda) v_\zeta + \frac{e}{c}\Lambda (R_0 + r) B_p \approx 0$$

$$\Rightarrow \Lambda \sim \frac{mc}{eB_p}\left(\cancel{\frac{r + R_0 + \Lambda}{R_0 + r}}\right) v_{th}\,\epsilon^{1/2}$$

$$q = \epsilon \frac{B_T}{B_p} \quad \Rightarrow \quad \Lambda \sim \frac{mc}{eB_T} v_{th}\, q\, \epsilon^{-1/2}$$

$$\boxed{\Lambda \sim \rho q \epsilon^{-1/2}}$$