# 2017 II : Q 1 Waves

a.) $\frac{\partial \tilde{n}}{\partial t} + n_0 \nabla \cdot \tilde{v} = 0$ $\qquad$ $m\frac{\partial \tilde{v}}{\partial t} = q\tilde{E} - \frac{\nabla P}{n} + \vec{\eta}$ ← $\vec{\eta}$ is force density ($\eta \sim \tilde{n}$)

$\downarrow \qquad\qquad \downarrow$

$\partial_t \tilde{n} + n_0 \nabla \cdot \frac{\partial \tilde{v}}{\partial t} = 0$ $\qquad$ $m\nabla \cdot \frac{\partial \tilde{v}}{\partial t} = q\,\underbrace{\nabla \cdot \vec{E}}_{\nabla \cdot \vec{E} = 4\pi q \tilde{n}} - \frac{1}{n_0}\nabla \cdot \nabla n T + \nabla \cdot \vec{\eta}$

$\Rightarrow$

$$\partial_t^2 \tilde{n} + \frac{n_0}{m}\Big(\underset{\omega_{p0}^2}{\underset{\equiv}{4\pi e^2}}\tilde{n} - \underset{3v_{th}^2}{\underset{\equiv}{\frac{T}{n_0}}}\nabla^2 \tilde{n} + \nabla \cdot \vec{\eta}\Big) = 0$$

$$\Rightarrow \boxed{\partial_t^2 \tilde{n} + \omega_{pe}^2 \tilde{n} - 3v_{th}^2 \nabla^2 \tilde{n} = -\nabla \cdot \vec{\eta}/m}$$

b.) Assume $\omega^2 = \omega_p^2 + c^2k^2$ $\Rightarrow$ $D = 1 - \frac{\omega_p^2}{\omega^2} - \frac{c^2k^2}{\omega^2}$

$\vec{\eta} = -n\nabla\phi = -n\nabla \frac{e^2|E_c|^2}{4m\omega^2}$ $\qquad$ need $|E_c|^2$ ...

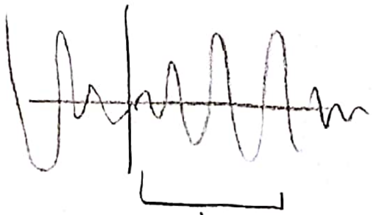
$$I \equiv \frac{|E_c|^2}{16\pi}\partial_\omega(D) = \frac{|E_c|^2}{16\pi}\left(2\frac{\omega_p^2}{\omega^3} + 2\frac{c^2k^2}{\omega^3}\right) = \frac{|E_c|^2}{8\pi}\frac{\omega^2}{\omega^3}$$

$$\Rightarrow |E_c|^2 = 8\pi I \omega \quad \Rightarrow \quad \eta = -ne^2\nabla\frac{2\pi}{m}\left(\frac{I\omega}{\omega^2}\right)$$

$$\Rightarrow \boxed{\eta = -\frac{\omega_p^2}{2}\nabla\left(\frac{I}{\omega}\right)}$$

c.) Unsure how to proceed?

$\omega_{beam} \sim \omega_{pe}$ $\Rightarrow$ beats :



↳ pulse length to maximize amplitude

d.) Kinetic equation for $F$: $\frac{\partial F}{\partial t} + \frac{dx}{dt} \nabla_x F + \frac{dk}{dt} \nabla_k F = 0$

Ray equations $\frac{dx}{dt} = v_g$, $\frac{dk}{dt} = -\frac{\partial \omega}{\partial x} = -\frac{\partial}{\partial x}(\omega_0 + \tilde{\omega}) = -i\underline{K}\,\tilde{\omega}$

$$\Rightarrow -\cancel{i}\Omega \tilde{F} + v_g \cancel{i}\underline{K}\tilde{F} - \cancel{i}\underline{K}\tilde{\omega}\nabla_k F_0 = 0 \qquad \frac{\partial}{\partial x} \to i\underline{K} \text{ (Langmuir wave number)}$$

$$\Rightarrow \boxed{\tilde{F} = -\frac{\tilde{\omega}\vec{\underline{K}} \cdot \nabla_k F_0}{\Omega - \underline{K} \cdot \vec{v}_g}}$$

$\tilde{\omega}$? $\qquad \omega^2 = \omega_{pe}^2\left(1 + \frac{\tilde{n}}{n_0}\right) = \omega_0^2 + 2\tilde{\omega}\omega_0 + \cancel{\tilde{\omega}^2}$ 2nd order $\qquad \omega_0^2 = \omega_{pe}^2$

$$\Rightarrow \boxed{\tilde{\omega} = \frac{\omega_{pe}^2}{2\omega_0}\frac{\tilde{n}}{n_0}}$$

e.) $\nabla \cdot \vec{\eta} = -\nabla \cdot \frac{\omega_p^2}{2} \nabla \int \frac{F_0 + \tilde{F}}{\omega_0 + \tilde{\omega}} d^3k = -\frac{\omega_{pe}^2}{2} \nabla^2 \int \frac{1}{\omega_0} \frac{F_0 + \tilde{F}}{1 + \tilde{\omega}/\omega_0} d^3k$

$$\approx -\frac{\omega_{pe}^2}{2\omega_0} \nabla_x^2 \int d^3k \,(F_0 + \tilde{F})\left(1 - \frac{\tilde{\omega}}{\omega_0}\right)$$

$$= F_0 + \tilde{F} - \frac{\tilde{\omega}}{\omega_0} F_0 - \cancel{\frac{\tilde{\omega}\tilde{F}}{\omega_0}} \to 0$$

$$= -\frac{\omega_{pe}^2}{2\omega_0} \int d^3k \left[-\underline{K}^2 \tilde{F} + \frac{F_0}{\omega_0} \underline{K}^2 \tilde{\omega}\right]$$

$$= \frac{\omega_{pe}^2 \underline{K}^2}{2\omega_0} \int d^3k \, \frac{\omega_{pe}^2}{2\omega_0} \frac{\tilde{n}}{n_0} \left(\frac{\underline{K} \cdot \nabla_k F_0}{\Omega - \underline{K} \cdot v_g} + \frac{F_0}{\omega_0}\right)$$

$$\Rightarrow -\Omega^2 + \omega_{pe}^2 + 3v_T^2 \underline{K}^2 = -\frac{\omega_{pe}^4 \underline{K}^2}{4\omega_0^2 n_0 m} \int d^3k \left(\frac{\underline{K} \cdot \nabla_k F_0}{\Omega - \underline{K} \cdot v_g} + \frac{F_0}{\omega_0}\right)$$