2017 I : Q2 GPP

Conservation of number density:

$$\int \left(\frac{df}{dt}\right)_c dv = -\nu \int f\, d\vec{v} + \nu \int f_M\, d\vec{v} \stackrel{?}{=} 0 \qquad u = -\frac{\Delta v}{\sqrt{2T/m}}$$

$$\int f\, d\vec{v} \equiv n \qquad \int f_M\, dv = \frac{n}{(2\pi T/m)^{3/2}} \int \exp\left(-\left(\frac{\Delta v}{\sqrt{2T/m}}\right)^2\right) d^3v$$

$$= n \frac{\cancel{(2T/m)^{3/2}}}{\cancel{(2T/m)^{3/2}}} \frac{1}{\cancel{\pi^{3/2}}} \underbrace{\int d^3u\, e^{-u^2}}_{\pi^{3/2}}$$

$$n - n = 0 \quad \checkmark$$

$\Rightarrow$ conservation of n.

Momentum

$$\int \left(\frac{df}{dt}\right)_c v\, d\vec{v} \stackrel{?}{=} 0 \qquad \int v f\, d\vec{v} \equiv \langle v\rangle \int f\, d^3v = n\langle v\rangle$$

$$\int v f_M\, d^3v \sim \int ((v - \cancel{\langle v\rangle}) + \langle v\rangle) \exp\left(-\frac{(v-\langle v\rangle)^2}{2T/m}\right) dv$$

$$\int \text{odd} \cdot \text{even} = 0$$

$$\rightarrow \int v f_M\, d^3v = \langle v\rangle \int f_M\, d^3v = n\langle v\rangle$$

$$\Rightarrow \int v f\, d\vec{v} - \int v f_M\, d^3v = 0 \quad \checkmark$$

Energy: $\int v^2 \left(\frac{\partial f}{\partial t}\right)_c dv \overset{?}{=} 0$

$$\int v^2 \exp\left(\frac{-(v-\langle v\rangle)^2}{2T/m}\right) dv = \int ((v-\langle v\rangle)+\langle v\rangle)^2 \exp(\quad)\, d\vec{v}$$

$$= \int (v-\langle v\rangle)^2 \exp(-\Delta v^2 a) + \xrightarrow{\text{odd}\,)(\text{even})} \cancel{2\,(v-\langle v\rangle)\langle v\rangle \exp(\,)} + \langle v\rangle^2 \exp(\,)\, d\vec{v}$$

$$\downarrow$$

$$= -\frac{\partial}{\partial a} \int d\vec{v}\, \exp(a(-\Delta v^2))$$

$$-\frac{\partial}{\partial a}\left[\left(\sqrt{\frac{\pi}{a}}\right)\right]^3 \qquad u = \sqrt{a}\,\Delta v, \quad dv = du/\sqrt{a}$$

$$= -\sqrt{\pi^3}\left(-\frac{3}{2} a^{-5/2}\right)$$

$$= \frac{3\sqrt{\pi^3}}{2}(2T/m)^{5/2}$$

$$\frac{n}{(2\cancel{\pi}T/m)^{3/2}} \frac{3\cancel{\sqrt{\pi^3}}}{2}(2T/m)^{5/2} \sim \frac{3nT}{m}$$

$$\langle v^2\rangle \int f_m\, d\vec{v} = n\langle v^2\rangle$$

$$\Rightarrow \int v^2 f\, dv = n\left(\frac{3T}{m} + \langle v^2\rangle\right)$$

to let $\int v^2 \left(\frac{\partial f}{\partial t}\right)_c dv = 0$ ✓